Changes in the descriptive frameworks of scientific theories

Abstract

It is shown that there-exist physical conditions for which scientific theories of countable numbers of objects are mathematically incomplete and that physically observable features can exist which cannot be derived within the scientific theory. However, this is just a descriptive problem arising due to restricting scientific theories to be in physically-real terms, and can be circumvented by the use of non-physically-real terms including the observed features. This is shown to give a derivation of Quantum Theory, and this form of incompleteness is also shown to be possible in scientific theories of living cells, ecosystems and the economies of nations. The impact on natural language descriptions of these systems is addressed.

0. Introduction

Although the initial target of Gödel's incompleteness theorems (Gödel [1931]) was the formulation of mathematics given in *Mathematica Principia* (Russell [1910-13]), the meta-mathematical proof can also apply other sufficiently related systems. Gödel's proof applies to any formal system $\boldsymbol{S}$ in language $\boldsymbol{L}$ based upon the set of axioms G_A which support logical deduction, arithmetic over the natural-numbers, and the creation of all the number-theoretic functions. The proof uses the arithmetic contained in the formal system $\boldsymbol{S}$ for a meta-mathematical approach within the scope of $\boldsymbol{S}$ that proves that the set of statements P_D derivable from G_A is less than the set of all true statements P within $\boldsymbol{S}$. This approach can also be applied to any scientific theory $\boldsymbol{S}$ with axiomatic basis P_A, such that if G_A is a sub-set of P_A, then the scientific theory $\boldsymbol{S}$ can be proven to be incomplete. Unfortunately, the direct pursuit of this approach encounters the difficulty that most scientific theories are not presented in axiomatic form, and the axiomisation of a scientific theory $\boldsymbol{S}$ can be highly non-trivial.

However, there is an alternative meta-science approach which looks at what constraints the underlying postulates of science place upon the unknown axiom set P_A of a scientific theory $\boldsymbol{S}$ in general. The mere act of writing down a scientific theory in the expectation that it can be used to make qualitative inferences and compute quantitative predictions in reality is implicitly based upon a number of assumptions. The first assumption is that there-exists some language $\boldsymbol{L}$ that can be used to write the scientific theory $\boldsymbol{S}$, such that logical inference and computation will be possible. This places the axiomatic requirements of mathematics upon the language $\boldsymbol{L}$, and those required for the incompleteness proof are itemised in section 1. However, this does not advance the search for incompleteness in a scientific theory because these requirements apply to the language $\boldsymbol{L}$ used for the theory $\boldsymbol{S}$, and not to the theory $\boldsymbol{S}$ itself; the fact that mathematics over the natural-numbers is proven to be incomplete does not imply that a theory $\boldsymbol{S}$ written in the language of mathematics is incomplete.

Assumptions are also made about the character of reality which a scientific theory $\boldsymbol{S}$ is intended to predict, where such assumptions have historically varied over time and between scientific disciplines. The postulates itemised in section 2 can be characterised as being among those of classical physics (specifically non-quantum physics) and are shared across the physical sciences. For the purposes of this paper, the set of postulates in section 2 will be taken to give a restricted definition of *scientific materialism*[1], where objects in reality are assumed to only change due to causal events involving other objects. The assumptions of *scientific materialism* given in section 2 will mean that every scientific theory $\boldsymbol{S}$ based upon them will be about how material causation results in changes which the theory $\boldsymbol{S}$

1 Terms used with a restricted meaning in this paper will be italicised throughout

seeks to predict. The term *scientific materialism* as used here roughly corresponds to the meaning of physicalism or materialism, but its usage is specifically being restricted to the issue of scientific description in scientific theories. When the incompleteness result is placed in a non-scientific context in section 10, the more general term *materialism* will be used instead.

The construction of a scientific theory ***S*** in language ***L*** and the framework of *scientific materialism* (§2) makes further assumptions about how to relate the terms of ***S*** to the objects of reality. The rules itemised in section 3 will be taken to define physically-real terms as a 1-to-1 notation of reality, which will give the theory ***S*** a form of *scientific realism* if the postulates of sections 2 and 3 are strictly adhered to. This restricted definition of *scientific realism* is about representation in scientific theories, where every term or operation given in physically-real terms in a theory ***S*** directly corresponds to some observable object or process in reality. Some of the meaning that may commonly be associated with the use of the term scientific realism is covered by the restricted definition of *scientific materialism* (§2). The terms *scientific materialism* and *scientific realism* are given these restricted definitions because their general usage is too ambiguous for the purposes of this paper. This gives the view of a scientific theory ***S*** as being:

1) written in a formal language ***L*** (§1)
2) based upon a general set of science postulates (§2)
3) having a set of correspondence rules (§3) that provide an empirical interpretation of ***S*** in terms of observable objects and processes
4) having a set of axioms specific to the physical system being modelled

This broadly corresponds to the view of a scientific theory given by the semantic approach (van Fraassen [1970]), but where the implicit assumption about a suitable language has been explicitly stated, and the axiomatic basis for ***S*** has been divided into the general metaphysical basis for science (2) and a system specific set (4).

In this view of a scientific theory ***S***, the postulates of *scientific materialism* (§2) and the rules of physically-real terms (§3) will guide the choice and expression of system specific axioms (4) in the construction of a scientific theory ***S***, such that it displays *scientific realism* and has the properties listed in section 4. This view is not being claimed to be universally applicable to all scientific theories, but to instead define a sub-set of scientific theories that fit this pattern. The reason for defining this sub-set is that the postulates of sections 2 and 3 will place restrictions upon the possible axioms of the unknown set of axioms P_A of any scientific theory ***S*** in this sub-set. This allows for a meta-science approach, where the postulates of sections 2 and 3 can be used to find the conditions for which the unknown set of axioms P_A of a scientific theory ***S*** must contain the axioms G_A required for Gödel's incompleteness proof. Now ***S*** will only be proven to be incomplete if ***S*** is known to be consistent, but if ***S*** is consistent then Gödel's incompleteness theorems (Gödel [1931]) also show that ***S*** cannot be proven to be consistent within ***S***. However, the postulates of *scientific materialism* (§2) include the assertion that reality is consistent, and the rules of physically-real terms (§3) define a 1-to-1 correspondence between ***S*** and reality. So if the *scientific materialism* of section 2 is true and a scientific theory ***S*** is in the strictly physically-real terms of section 3, then the *scientific realism* of ***S*** (§4) will mean that it is known to be consistent.

This meta-science approach does not seek to find the axioms P_A of some scientific theory ***S***, but defines a sub-set of scientific theories that will be guaranteed to possess an axiomatic basis of some form, and then finds the physical conditions for which P_A will have to contain G_A in order for ***S*** to possess *scientific realism* (§4). Gödel's incompleteness theorems (Gödel [1931]) further prove that extending an incomplete theory with additional axioms of the same form as those in G_A will not alter the incompleteness proof. So once the axioms G_A are shown to be a sub-set of the unknown set of axioms P_A of some scientific theory ***S***, the additional axioms of P_A will not change the incompleteness result as long as they are of the same form as those of G_A. This will be true for the additional unknown axioms of P_A if they comply with the postulates of *scientific materialism* (§2) and the rules of *p*hysically-real terms (§3), which are required for the scientific theory ***S*** to retain *scientific realism*. In this way, a scientific theory ***S*** with unknown axiomatic basis P_A can nonetheless be proven to be incomplete by the requirement of *scientific realism* as defined in section 4.

For the sub-set of scientific theories that fit the view given above, other concepts arising in the philosophy of science will also be given restricted definitions for the purposes of this paper. The process of deriving the elements of P_D from the axiomatic basis P_A, and experimentally testing whether they predict all the statements of observations P_O expressed in the language ***L***, can be viewed as constituting *normal science research*. Finding an element p of P_O which is not an element of P_D would technically falsify the scientific theory ***S***, in the manner discussed by Popper ([1935]). However, not all such falsifications will be equal. It may often be possible for such a statement $p \in P_O$, $p \notin P_D$ to be accounted for in a modified theory ***S'*** where an existing axiom is altered and/or axioms added such that p can be derived and $P_O \subset P_D$. In the exceptional case, the observation p could be labelled as being an anomaly in ***S'***, and the statement p directly added to P_A as an exceptional axiom. Such successful theory modification can be viewed as constituting the practise of *normal science progress*. This leaves the cases where one or more such observations ($p \in P_O$, $p \notin P_D$) cannot be accounted for by modifications to the axiomatic set P_A of an established scientific theory ***S***, and instead requires a totally new axiomatic set. Such a discontinuous jump in axiomatic basis will be viewed as giving the basis for the scientific *paradigm shifts* of Kuhn ([1962]).

1. Formal Systems

This paper will have a number of points of comparison with Wittgenstein's *Tractatus* ([1921]) through the underlying theme being the use of language to describe reality: through picture theory in *Tractatus*; and in terms of physically-real scientific theories here. There will arise some points of comparison, but the *Tractatus* starts with a very significant point of disagreement:

> 1.1 The world is the totality of facts, not of things

The view of *scientific materialism* (§2) is essentially the other way round, with the material things of the world being viewed as primary and facts being stated about those things. In spite of the primacy of things in the scheme of science, it first assumes that there-exists a suitable formal language ***L***, which is why the structural points required for the language ***L*** will be stated first. Issues with regards to the philosophy of mathematics will be ignored here; the points given are mostly just those required for the identification of Gödel's 'related systems'. A relaxed view of symbolic notation is adopted because the comparison of axioms will be in functional terms that are independent of the notation used.

1.1 Logical terms A, B, ... etc. and logical operations between the terms, A and B, A or B, not-A.

Such terms will be required to denote objects in reality.

1.2 Predicates: $P(object) \rightarrow value$.

Such predicates will be required to denote the measurement of the property values of objects.

1.3 Logical implication: if A and A implies B, then B.

This is the basic form of inference required for logical deduction within the language ***L***.

1.4 Sets with types, and cardinal numbers defined over the sets.

A predicate P can be used to define the classification of objects into a set where all the elements possess the same value for the predicate P. It must be noted that the set theory required for science will be required to distinguish between *urelements* denoting objects, e.g. A, and sets of objects, e.g.

$\{A, ...\}$, as the two are inequivalent in reality. The use of types will have the effect of preventing set-type paradoxes from occurring in scientific language.

1.5 Arithmetic over the cardinal numbers.

The set theory must support the addition and removal of new elements to a set, and this defines the successor $s(n) \rightarrow n+1$ and predecessor function $p(n) \rightarrow n-1$ over the cardinal numbers of the sets. A repeat the successor or predecessor functions a specific number of times N will define addition and subtraction respectively. Multiplication can then be defined as an N repeat of addition, and in this way give the operations of arithmetic over the natural-numbers.

1.6 Logical induction: if $P(0)$ and $P(s(n))\ \forall n$, then $P(n)\ \forall n$.

Induction over the natural-numbers is specifically required for Gödel’s incompleteness theorems, and so this form of induction will be required to be present within a scientific theory for Gödel’s incompleteness proof to be applied.

1.7 A set of axioms P_A can be used to derive a set of deduced statements P_D expressed in the language $\boldsymbol{L}$.

The condition that $\boldsymbol{L}$ is a deductive language will be required for the intention of science to make predictions of new facts about reality from some finite basis of knowledge.

1.8 Initial functions ($s(n)$, $p(n)$, $z(n)$, P_i).

An n-repeat of the predecessor function $p(n)\ \forall n>0$ will define the zero function $z(n) \rightarrow 0$, and the projection functions $P_i(x_1, \ldots, x_m) = x_i\ \forall x_1, \ldots, x_m$ can be defined in terms of predicates (1.2).

1.9 Number-theoretic functions $f(x)$ over natural-number valued variables x.

With the initial functions defined in 1.8, all number-theoretic functions can be generated starting from them by the use of the function creation rules of substitution and recursion (Mendelson [2010])). Function f is obtained from functions g, $h_1, \ldots, h_m$ by substitution when:

$$f(x_1,..., x_n) = g(h_1(x_1,..., x_n),\ldots,h_m(x_1,..., x_n))$$

Function f is obtained from functions g and h by recursion when:

$$f(x_1,..., x_n, 0) = g(x_1,..., x_n)$$

$$f(x_1,..., x_n, y+1) = h(x_1,..., x_n, y, f(x_1,..., x_n, y))$$

Such functions will be used in scientific theories to make numerical predictions of the measured values of properties of objects.

1.10 Gödel ([1931]) proved that for a consistent formal system $\boldsymbol{S}$ possessing the properties 1.1-1.9 there-exists a set P_U in the set of statements P expressed in the formal language $\boldsymbol{L}$ which cannot be deduced, i.e. $P_U \neq \{\}$ and $P_U \cap P_D = \{\}$.

There are two important features about the proof which must be carefully noted in order to show that it applies to formal systems other than that of mathematics. The first is its meta quality, where the arithmetic of the formal system $\boldsymbol{S}$ is used to perform the arithmetic of the proof within the context of the formal system $\boldsymbol{S}$ itself. For a scientific theory, this will mean that the operations of arithmetic must be present within the scientific theory itself, and not just in the mathematics used to manipulate the elements of the theory.

The second point to note about Gödel’s proof is that it includes a universality condition: that every number-theoretic function must be expressible within the formal system $\boldsymbol{S}$. This is because the proof uses induction over the natural-numbers (1.6) to show that the Gödel number g of some specific statement u is not within the set of Gödel numbers of the derivable statements P_D. This universality feature will also be required to be present within the details of a scientific theory in order for the theory to qualify as one of Gödel’s 'related systems'.

2. Scientific Materialism

The construction of formal theories in science for the purposes of predicting the results of experiments is based upon a number of assumptions about reality, of which the key ones for the purposes of this paper are given here. Although the various justifications for the underlying assumptions of science vary significantly in quality, the focus of this paper is strictly on the use of formal language for deduction in scientific theories, and from this perspective all such justifications share exactly the same logical classification: they are not deduction. The underlying issue being the problem of induction, and all the assertions given here either contain this problem explicitly by using the words 'all' and 'every', or by implicitly assuming universal applicability. This problem of induction has to be carefully addressed here because Gödel’s incompleteness proof critically depends upon the use of logical induction within the scope of a formal system, and so logical induction within a scientific theory is required to be logically valid for the purposes of this paper.

Explicitly, the problem is that induction from a statement p_n about a finite set of empirical observations P_O of size n to an assertion p_∞ about the set of all possible observations is not logically valid. The way that the formal system specified in section 1 achieves valid logical induction (1.6) over the natural-numbers is through the use of the generator of the natural-numbers, the successor function (1.5). Such use of generators is one of the most significant approaches used in the construction of a scientific theory $\boldsymbol{S}$ such that induction within the scope of $\boldsymbol{S}$ is logically valid. Of course the problem of induction hasn't gone away, it has just been transferred from the set P_O to the generator g of the set. A finite set of observations P_O may allow a generator g_n to be logically inferred for the set, but the induction of g_n to be the generator g_∞ of all possible observations isn't logically valid. If the inferred generator g_n is the same for all the elements of P_O, this provides a very good justification for thinking that g_∞ will be the same as g_n, but this still isn't valid logical induction. This problem can be 'resolved' by asserting the generator g_∞ in the axiomatic basis of the theory, and this is the tactic used in mathematics where the successor function $s(n)$ is part of the axiomatic basis of arithmetic over the natural-numbers. However, the problem of induction still hasn't really gone away, it has now just been transformed into an issue of falsification. The assertion of a universal generator g_∞ for all possible observations can now be directly falsified by a single observation that cannot be generated using g_∞, giving instances that are in strict accordance with Popper ([1935]) on the role of falsification. Such an assertion of a universal generator g_∞ underlies Relativity in physics, where one and only one example of an empirical observation of a speed in excess of the speed of light is sufficient to falsify Relativity, because such an observation would falsify the underlying universal generator g_∞ of Relativity.

For the purposes of this paper, this generator approach is implicitly contained within assertion 2.8 that the countable numbers of objects of different types only change due to object conversion reactions, as this gives realisations of the predecessor function $p(n)$ and successor function $s(n)$ over the countable numbers of objects of different types. This will mean that logical induction of the form given in 1.6 will be logically valid within the scope of a scientific theory $\boldsymbol{S}$ based upon the assertions given here, which is all that will be required in this paper.

These logical difficulties with constructing a scientific theory on the basis of empirical observation can be more succinctly expressed when the scientific theory $\boldsymbol{S}$ has an identifiable axiomatic basis P_A. In this case, the practise of *normal science research* provides direct tests between the predictions contained within the set of derived statements P_D of $\boldsymbol{S}$ and the set of empirical observations P_O. If discrepancies arise between P_O and P_D, the practise of *normal science progress* includes methods by which the set of observations P_O can be used to logically infer how to modify the axiomatic basis P_A of $\boldsymbol{S}$ in order to change the set P_D so that it encompasses P_O. This is most obvious for the anomaly approach to discrepancies, as the anomalous observation is simply added to the axioms P_A. It is also possible that the required modifications to the axioms of P_A can be logically inferred from empirical observation, but it can be a different matter when it comes to the insertion of new axioms or the total change in axiomatic basis of a *paradigm shift*. The issue being that the set of statements P_D can be derived from the axioms P_A, but the axioms cannot be logically derived, which is why they have to be declared as axioms. This would not be a problem for science if the set of axioms P_A could be logically inferred from the set of empirical observations P_O, but this almost inevitably encounters the problem of induction. So in strictly logical terms, this means that the practise of science does not contain methods by which the axioms of science can be logically inferred. This limitation of science when it comes to its own axioms provides a basis for some agreement with the views of Feyeraband ([1975]) about the lack of universal methods in science. Furthermore, if the strict use of logical inference takes a prominent position in the definition of science, then the resulting demarcation line would in effect separate science from its own axioms.

With this point of view, the assertions of this section are metaphysics. The circularity of using the results of a scientific theory in justification of its axiomatic basis, and the problem of induction, can both be viewed as being broken by metaphysical assertions of science axioms. No matter how convincing a justification for science axioms may seem, ultimately they are just asserted. So this paper will adopt a straightforward approach, and simply present the following naked assertions as being the metaphysical basis of *scientific materialism* required for the purposes of this paper.

2.1 There-exists an objective reality that can be reliably and repeatedly measured.

The underlying assumption of science, in that if it wasn't true the pursuit of science would then appear to be somewhat pointless. All of reality would have to be measured for this assumption to be technically verified, and only then it would no longer be metaphysical.

2.2 All the features of reality can be measured.

Perhaps the defining metaphysical assumption of scientism. The universality of the word 'all' means that the assumption would only have been verified when every feature of reality had been measured.

2.3 There-exist material objects that can be classified into different types.

As was noted in 1.4, this will require a set theory in the descriptive language $\boldsymbol{L}$ of science with a system of types to distinguish between objects and the sets into which they can be classified.

2.4 All material objects are composed of fundamental (atomic) objects.

Finding the fundamental objects of all material objects is assumed to provide the basis for deriving all the properties of material objects. Again, the universality contained in 'all' implies that this would not have been technically verified until every material object had been measured.

2.5 Reality is consistent.

The most important part of the consistency of reality for this paper will be the condition that no object can both exist and not exist at the same time (considerations of Quantum Theory will be shown to be irrelevant in §8).

2.6 There-exist events in reality such that the measured values of observations only change when caused to do so by these events.

It is the denotation of these causal events which will give all the operations contained within a scientific theory. For the cases being considered in this paper, this will mean that the operations of arithmetic over the numbers of objects in some physical system must be implemented as causal events by the objects of the physical system itself. If this is not the case, then the operations of arithmetic will be in mathematics only and not within the scope of the scientific theory itself. Consequently the proof of incompleteness can then not be constructed solely within the scope of the scientific theory, and so the theory will not be proven to be mathematically incomplete.

This paper will demonstrate in §8 that this principle of strict material causation is not challenged by Quantum Theory, by deriving Quantum Theory on the basis of the assertions of this section.

2.7 Logical inference over observations is valid.

This is required for the purposes of science, where the assumption is also made that the formal system of mathematics provides a valid deductive language for describing reality (§1).

2.8 There-exist object conversion reactions where objects are caused to change object type.

This is the critical feature which will be required for the incompleteness proof of §6.

2.9 The numbers of objects does not spontaneously change.

The scientific theories of interest here will be those where the numbers of differently classified objects are causally changed by object action. This is an important part of strict material causation, and it is explicitly given here so as to rule out object numbers changing spontaneously without a cause that can be traced back to an object in reality.

2.10 Relativity principles explain how measured values can change without causal events.

Such Relativity principles are technically required in order to have a consistent framework for science where measurements of reality only change due to the causal events of 2.6. Relativity can be viewed as requiring further metaphysical assumptions about time, space and motion, but these will have no direct impact on the proof when 2.8 is true. However, the Relativity principles do lead to two features of reality that are relevant to the physical conditions required for the incompleteness proof of §6, but not for the actual mathematics of the proof.

2.10.1 Energy postulate.

There-exists an energy measure of objects such that the total energy is always conserved.

2.10.2 Wave motion.

The application of the Relativity principles to time and space lead to wave equations where such wave motion is classified as being distinct from particle motion. It can be noted that for gravity waves and electromagnetic waves, the question of what is actually being subject to wave motion is given the answer of the 'vacuum' in physics, where the 'vacuum' has a metaphysical character.

3. Physically-Real Terms

With the basis for the formal language ***L*** used by science given in section 1, and the core metaphysical basis of *scientific materialism* given in section 2, the rules for physically-real terms can now be given.

3.1 A physically-real term is a strictly 1-to-1 bi-directional denotation of an experimentally measurable feature of reality in some formal language ***L*** (§1).

3.1.1 Reality→notation: this allows for the construction of scientific models of the physical features measured in experiments.

3.1.2 Notation→reality: this allows scientific models to be used to make successful predictions of the measurements of future experiments.

3.2 Physically-real predicate $P(object) \rightarrow value$.

In this notation, the predicate P (1.2) is denoting a process of experimental measurement, where the value of the predicate gives the experimentally measured result.

3.3 A physically-real predicate P can be used to classify objects into a set.

This gives the definition of a *well-defined* set in physically-real notation, where the set contains *urelements* of a different set-theoretic type from sets (1.4) and an *urelement* is physically-real term denoting an object.

3.4 A physically-real term or proposition P is true if the object state it denotes exists, and false if it doesn't exist.

This is logical truth values strictly given by existence, so when this physically-real notation is used for denoting a consistent reality (2.5) the consistency of reality will be transferred to a formal system $\boldsymbol{L}$ strictly based on physically-real terms.

3.5 Logical operations (1.1) on physically-real terms also have logical truth values given by existence.

3.5.1 Logical-negation: not-P says that P doesn't exist.

3.5.2 Logical-and is given by conditional existence: A and B exist.

3.5.3 Logical-or is given by alternate existence: A or B exist.

3.6 An operator notation denotes object changes produced through causal events (2.6): A → B.

Such operators denoting causal events are the only operators acting within the scope of a scientific theory. All other mathematical operators reside within the scope of mathematics ($\boldsymbol{L}$) and not within the scope of a scientific theory $\boldsymbol{S}$, and so will be unavailable for the purposes of Gödel's meta-mathematical proof of incompleteness.

4. Scientific Theories

The metaphysical postulates of section 2 define *scientific materialism*, such that strictly adhering to the rules of section 3 for physically-real terms will result in a scientific theory $\boldsymbol{S}$ that possesses *scientific realism*. This section itemises such a construction for a generic theory $\boldsymbol{S}$ to which system specific axioms can then be added to give an axiomatic scientific theory $\boldsymbol{S}$ of some physical system.

4.1 A physically-real scientific theory is a formal model $\boldsymbol{S}$ in some formal language $\boldsymbol{L}$ that is based upon denoting the reality of some physical system in strictly physically-real terms (§3).

The scientific program assumes that it will always be possible to construct such a formal model $\boldsymbol{S}$ of a physical system.

4.2 A scientific theory $\boldsymbol{S}$ in strictly physically-real terms with physically-real operator causation (3.6) will be consistent for a consistent reality (2.5).

Scientific models must be consistent so that their predictions of reality are reliable and definite.

4.3 Qualitative and quantitative predictions of experiment measurements can be inferred or calculated using the formal scientific model $\boldsymbol{S}$.

In order to make accurate quantitative predictions for experimental measurements, the scientific model $\boldsymbol{S}$ will be required to use computable functions which will necessarily take the form of partially recursive number-theoretic functions. The significance of this for the purposes of this paper is that all such functions can be derived from the set of initial functions (1.8) by the application of the function creation rules of substitution and recursion (1.9).

4.4 Verification or falsification of the scientific predictions made by $\boldsymbol{S}$ for experimental results can be used to partially verify or falsify the scientific model $\boldsymbol{S}$.

For a consistent reality (2.5) and scientific theories constructed in physically-real notation (4.1) any inconsistency with experimental measurements for some scientific theory of a physical system necessarily implies that the scientific theory has not been correctly constructed.

4.5 Physically-real implication: If A exists and A is the cause of B then B exists.

This gives the basis for scientific deduction of the future measured values for some object state caused to exist by the current object state. The time scale between the two causally related states is not given because it will not directly impact the incompleteness proof.

4.6 Physically-real induction: If physically-real predicate P is true for a set containing one *urlement*, and it is always true for the successor function operating over the set, then the predicate P is true for the set no matter how many *urlements* it contains.

This is is logical-induction for $n>0$, which differs from the mathematical induction of $n \geq 0$, as it does not include the unique empty set {}. This difference is because spontaneous object creation for every different type of object does not occur (2.9), which is what would be required to be true for logical-induction from $n=0$.

4.7 A physically-real scientific theory $\boldsymbol{S}$ is a consistent formal system in some formal language $\boldsymbol{L}$ that supports deduction, arithmetic and computation.

The operation of deduction will be within the scope of all physically-real scientific theories because inference is given by modelling causation (4.5). This is a critical feature of using physically-real terms and gives the basis for *scientific realism*. In contrast, the operations of arithmetic and computation will only be within the scope of the scientific theory if they are denoting causal events that implement these operations over the sets of objects. When these operations are not within the scope of the scientific theory itself, they will still be available through the mathematical basis of the formal language $\boldsymbol{L}$ used for scientific theories.

5. Physically-Real Arithmetic

The basis for *scientific realism* in a scientific theory $\boldsymbol{S}$ has been given in the preceding sections by the condition that the theory should be strictly expressed in the scientific language of physically-real terms. We will now start to address the physical conditions required for the set of axioms P_A of the theory $\boldsymbol{S}$ to contain the set of axioms G_A as a sub-set. This first requires considering the physical conditions under which the operations of arithmetic will be expressed in physically-real terms within the scope of scientific theory $\boldsymbol{S}$. The arithmetic in question will solely be in terms of the numbers of objects in some physical system that are classified into different sets by physically-real predicates.

5.1 Object type conversion reactions (2.8) of the form: $A \rightarrow B$.

The two types of object denoted A and B will be classifiable into *well-defined* sets by physically-real predicates that identify the two types of object, and each set will have a cardinal number giving the number of *urelements* denoting the number of objects in reality.

5.1.1 Successor operation: $s(n) \rightarrow n+1$.

This will be a physically-real operation in any scientific theory of the object reaction because each conversion $A \rightarrow B$ increments the number of B type objects by 1.

5.1.2 Predecessor operation $p(n) \rightarrow n-1$.

This will be a physically-real operation in any scientific theory of the object reaction because each conversion $A \rightarrow B$ decrements the number of A type objects by 1.

5.1.3 Zero operation $z(n) \rightarrow 0$.

If the object conversion $A \rightarrow B$ occurs for every A type object in any initial collection of A type objects, then the object conversion process will decrease any number n of A type objects down to 0. This means that the operation of the zero function $z(n)$ in a scientific theory of this object conversion process will be physically-real.

5.1.4 Projection functions P_i

These can be constructed in physically-real terms for physically-real predicates (3.2)

5.2 N-repeat causal control C_N of another operation.

The operation of addition will be physically-real when some number n of objects in a particular set has a specific number N of further objects added to the set: $n \rightarrow n{+}N$. The specificity of the number N added will necessarily require some form of control, which by *scientific materialism* (§2) can only be implemented by some type of control object C_N.

5.2.1 Addition through N-repeat control of the successor operation 5.1.1, $C_N(s(n)) \rightarrow n + N$.

5.2.2 Multiplication through N-repeat control of arithmetic operation 5.2.1, $C_M(C_N(1)) \rightarrow M{\times}N$.

5.3 A non-thermal energy source is required to power the arithmetic operations of 5.2.

The energy measure (2.10.1) and entropy measure (defined as the configurational entropy over the distribution of objects into classification sets) of physics imply that the additive operation 5.2.1 due to some control object $C_N(n) \rightarrow n{+}N$ will require an energy source on two counts. Firstly, a control process would necessarily do work (follows from §2) and so require an energy source, and secondly an additive process over object numbers will lower the entropy of an object system and so cannot occur without a source of energy. The detailed consideration of entropy in thermodynamics reveals that such arithmetic processes will specifically require a non-thermal energy source, but such details will not impact the incompleteness proof given in §6.

6. Incompleteness

Although Gödel's incompleteness theorems can be briefly described as applying to formal systems that contain full logical deduction and arithmetic over the natural-numbers, the detail of the incompleteness proof is dependent upon every possible number-theoretic function being present within the formal system ***S*** (1.10). This means that a physical system that implements the object arithmetic of section 5 will be insufficient for a physically-real scientific theory of the system to be one of Gödel's 'related systems'.

Number-theoretic functions will occur in scientific theories as terms which model the causal processes by which the numbers of objects of different types are changed, such as the object conversion process of 5.1. So in any physical system with a finite number of physical processes that change the classification types of objects, the number of number-theoretic functions directly occurring in a scientific theory will be strictly finite. This means that the universality condition referred to in 1.10 will not be realised in physically-real terms within the scientific theory, and consequently it will not be one of Gödel's 'related systems'. However, it is possible for a physical system to exist where recursive number-theoretic functions can be generated within the scope of a physically-real scientific theory ***S*** when both the initial functions ($s(n)$, $p(n)$, $z(n)$, P_i) and the function creation rules (substitution, recursion) are expressed as physically-real operations within the scientific theory. When this is the case, those recursive number-theoretic functions not directly occurring within the scientific theory, could nonetheless be generated in physically-real terms within the scope of the scientific theory. If the physical system is such that this function creation process will always be possible, then the universality condition of 1.10 will be satisfied and the physically-real scientific theory ***S*** of the physical system will be one of Gödel's 'related systems' which is proven to be incomplete. This section details when Gödel's incompleteness proof can be applied to a scientific theory ***S*** of a real physical system.

6.1 Formal deductive system ***S*** expressed in a formal language ***L***.

6.1.1 A scientific model ***S*** in formal language ***L*** (§1) is defined by the use of physically-real terms (§3) to denote objects (2.3) and object causation (2.6) on the states of other objects.

6.1.2 The scientific model ***S*** is consistent because it is based upon a calculus denoting a consistent reality (2.5) in physically-real terms; it is consistent because reality is consistent (4.2).

6.1.3 The scientific model $\boldsymbol{S}$ is a deductive system because the causal events (2.6) denoted in physically-real terms give rules of implication (4.5) and induction (4.6) within $\boldsymbol{S}$.

6.2 Includes arithmetic over the natural-numbers.

6.2.1 The classification of objects by physically-real predicates (3.2) gives *well-defined* sets (3.3) with cardinality given by the natural-numbers.

6.2.2 A scientific model $\boldsymbol{S}$ of a physical system meeting the physical conditions given in §5 will include the denotation of the operations of arithmetic within $\boldsymbol{S}$ itself.

6.3 Partial recursive number-theoretic functions $f(x)$ over natural-number valued variables x.

6.3.1 The variable x denotes the number of objects classified into a particular *well-defined* set (3.3).

6.3.2 The domain of the variable x and function f is restricted to the physical system for which the scientific model $\boldsymbol{S}$ is proposed to hold, and so every function f is partial.

6.3.3 Every function f in $\boldsymbol{S}$ models object causation (2.6) in the physical system so as to make predictions of the future number of objects in some object classification set. This requirement of computable functions ($f(x)$) over natural-number valued variables (x) means that every function f must be a partial recursive number-theoretic function (4.3).

6.4 Every partial recursive number-theoretic function $f(x)$ (1.9) can be expressed within the scope of $\boldsymbol{S}$.

6.4.1 The initial functions (1.8) are present within $\boldsymbol{S}$ for a physical system that satisfies the physical conditions of §5.

6.4.2 Substitution: function f is obtained from functions g, $h_1,\ldots, h_m$ by substitution (Mendelson [2010]) when:

$$f(x_1,..., x_n) = g(h_1(x_1,..., x_n),\ldots,h_m(x_1,..., x_n))$$

The variables $x_1,..., x_n$ will denote the number of objects classified into n sets in $\boldsymbol{S}$ and the functions h_i will denote m causal actions which change the objects into different types. This notation specifies some functional unit of objects (denoted by h_i) which takes a number of objects of n different types (denoted by $x_1,..., x_n$) to generate some number of objects of a different type (this action is denoted by $h_i(x_1,..., x_n)$). The given function substitution rule would then denote the object output of these functional units being input into another functional unit denoted by g. The net action on object numbers given by this hierarchy of object conversion processes would then be denoted by the function f as defined. This gives the basis for the function substitution rule to be physically-real terms as a tree-like cascade of object conversion reactions which feed into each other.

6.4.3 Recursion: function f is obtained from functions g and h by recursion (Mendelson [2010]) when:

$$f(x_1,..., x_n, 0) = g(x_1,..., x_n)$$

$$f(x_1,..., x_n, y+1) = h(x_1,..., x_n, y, f(x_1,..., x_n, y))$$

The meaning of the variables and functions is the same as for 6.4.2, where the new natural-number valued variable y must denote the number of objects in a new object classification set defined by some physically-real predicate. This addition of the variable y will only be in physically-real terms in $\boldsymbol{S}$ if the physical system generates new types of object that were not previously present in the system. The recursion rule then requires a step-wise operation where the next step of the object conversion process is dependent upon the previous step, which it will be for discrete objects (2.4) subject to individual causation (2.6).

6.4.4 Universality: the function creation rules 6.4.2 and 6.4.3 must always be applicable in strictly physically-real terms for all possible states of the physical system.

For the function creation rules to be repeated indefinitely some of the newly created object types (with variable y denoting their numbers) must be input into some pre-existing object conversion processes. This condition is so that the outputs of functions created by recursion (6.4.3) then form part of further function creation by substitution (6.4.2). In physical terms, this will require a physical system of the form of a indefinitely growing network where new nodes are added and linked into the network by the action of the physical system itself.

6.5 The Gödel number g can be constructed for any expression P expressed within $\boldsymbol{S}$ because $\boldsymbol{S}$ contains both arithmetic over the natural-number valued variables x and every partial recursive number-theoretic function $f(x)$ (6.4).

6.6 The diagonal function D giving the Gödel number g of an expression $P(g)$ taking as argument the Gödel number g of P itself can be constructed within the scope of $\boldsymbol{S}$ by 6.4 and 6.5.

6.7 Both the Gödel sentence G and Rosser sentence R can be constructed within the scope of $\boldsymbol{S}$ by the above conditions, to give 2 known elements of a set P_U of undecidable propositions. Gödel's meta-mathematical proof can be constructed within the scope of $\boldsymbol{S}$ and so the scientific model $\boldsymbol{S}$ for the physical system meeting the required conditions is proven to be mathematically incomplete.

6.7.1 The scientific model $\boldsymbol{S}$ is known to be incomplete because *scientific realism* has ensured that $\boldsymbol{S}$ is known to be consistent for a consistent reality (2.5). So Gödel's proof that an incomplete theory $\boldsymbol{S}$ cannot be proven to be consistent within itself is irrelevant, as $\boldsymbol{S}$ is known to be consistent by other means.

6.7.2 There is no hidden variable theory $\boldsymbol{T}$ which can be constructed from $\boldsymbol{S}$ by the addition of a further natural-number valued variable x_h such that $\boldsymbol{T}$ is mathematically complete.

This is because the universality condition 6.4.4 ensures that every possible natural-number valued variable y has already been included in the course of the incompleteness proof. This means that the hidden variable theory $\boldsymbol{T}$ has already been considered in proving $\boldsymbol{S}$ to be incomplete, and so $\boldsymbol{T}$ has also been proven to be incomplete.

7. Describe Another Way

The incompleteness result given in section 6 might lead to an expectation that this section would contain a similar proposition to that of section 7 in Wittgenstein's *Tractatus* ([1921]):

> 7. What we cannot speak about we must pass over in silence

However, this is not what the incompleteness result of section 6 means. Although it may be tempting to dismiss the proven existence of undecidable propositions as having any scientific relevance, this ignores the fact that strict adherence to *scientific realism* means that the undecidable propositions will be expressed in physically-real terms. The bi-directional character of the physically-real notation of section 3 implies that these undecidable propositions in theory can correspond to physical states in reality (3.1.2). So an undecidable proposition u would be expressed in the physically-real terms of scientific language $\boldsymbol{L}$ within the scope of a scientific theory $\boldsymbol{S}$, meaning that we would be able to speak about it. The issue stemming from the incompleteness of section 6 is that it isn't possible to derive u within the scientific theory $\boldsymbol{S}$, and so we would not be able to derive what we could speak about in the very same terms.

The philosophical framework of *scientific materialism* (§2) implicitly includes within it the reductionist assumption that it will always be possible to describe observations of reality in the strictly physically-real terms of *scientific realism*, which is still true here. However, the postulates of section 2 do not guarantee that this process will be reversible, and the incompleteness result of section 6 claims that examples of such irreversibility can exist in reality: there can exist observations of some physical systems which are due to the physical basis found by reductionism, but the actual statements of observation cannot be derived from that basis. The known undecidable propositions (e.g. the Gödel and Rosser sentences) can be characterised as being self-referential, which through logical implication in a scientific theory $\boldsymbol{S}$ being realised due to modelling causation in reality (4.5), could only correspond to a state of causal closure in a physical network system (by 6.4.4).

Despite the incompleteness proof of section 6, it is nonetheless possible to construct a formal model $\boldsymbol{S_C}$ in which undecidable propositions can be incorporated, such that $\boldsymbol{S_C}$ is both consistent and complete. Gödel's incompleteness theorems (Gödel [1931]) prove that this would not be possible in physically-real terms denoting the numbers of objects in different classification sets as natural-numbers, and so the adherence to physically-real terms (§3) will have to be dropped in order to construct such a complete theory $\boldsymbol{S_C}$. This necessity of having to change the descriptive framework can be expressed in a form corresponding to that of Wittgenstein as:

> What we cannot derive in deductive language using only physically-real terms we must describe another way

This section outlines this 'other way' and the consequences of such a change in descriptive framework.

7.1 Assume that there-exists causal closure within a physical network system that meets the required conditions of §6 and can be observed to possess one undecidable property p.

Such a physical system with an undecidable property p is not guaranteed to exist at this point, and so we will simply assume that such a physical system exists. The purpose of this assumption is to consider what steps would be required to transform the incomplete physically-real scientific theory $\boldsymbol{S}$ into a complete theory $\boldsymbol{S_C}$.

7.2 Add the observed undecidable property p to the scientific theory $\boldsymbol{S}$ to give a modified theory $\boldsymbol{S'}$.

Although this may appear to be straightforward, the undecidable property p has been assumed to be a property of a causally closed arithmetic state of a physical network system (7.1). This means that the property p is not being attached to an *urelement* denoting a single object, but being attached to a collection of *urelements* forming a causally closed state in the physical system. As such a collection of *urelements* will be denoted as a set of objects in $\boldsymbol{S}$, the property p is being attached to a set in the modified scientific theory $\boldsymbol{S'}$. If this set is denoted s and the undecidable property p attached to it, this will give a set-theoretic type conflict for s in the scientific theory $\boldsymbol{S'}$ because a set is not strictly a physically-real term in $\boldsymbol{S}$; it is the *urelements* inside a set which are the physically-real terms denoting real physical objects. The implicit reductionism assumption of 2.4 is that every observation of reality can be reduced to the properties of one or more objects inside a set denoted in a scientific theory. When this reductionism is true, the type distinction between *urelements* and sets will be true within $\boldsymbol{S}$. However, when an undecidable property p is attached to a set s, such reductionism is not possible because p cannot be derived from the state of the objects inside the set s, and so s has the status of an *urelement* in the scientific theory $\boldsymbol{S'}$. This is a set-theoretic type conflict and indicates that this form of modified theory $\boldsymbol{S'}$ will not be acceptable in the language $\boldsymbol{L}$ (§1).

7.3 Change the physically-real terms $x_{(n)}$ denoting the numbers of objects in $\boldsymbol{S}$, into non-physically-real terms $x_{(r)}$ denoting the numbers of objects in terms of real-numbers in $\boldsymbol{S_C}$.

Gödel's incompleteness theorem only applies to formal systems including arithmetic over the natural-numbers, not over the real-numbers, and so the incompleteness proof of §6 will not apply to scientific theory $\boldsymbol{S_C}$. The undecidable property p can also be attached to a non-physically-real term s such that the theory $\boldsymbol{S_C}$ can be complete. There will not be a set-theoretic type conflict in this case because the real-number valued non-physically-real terms $x_{(r)}$ aren't denoting the cardinality of sets. They are instead giving real-number valued estimates for the numbers of objects of particular types, and so the theory $\boldsymbol{S_C}$ doesn't contain the sets which were causing the set-theoretic type conflict.

7.4 In order to extract predictions for the physically-real values $x_{(n)}$ actually measured in experiment, a mathematical conversion function will be required such that $x_{(n)}=\boldsymbol{M}(x_{(r)})$.

This conversion process from the real-number values of $x_{(r)}$ to the natural-number values of $x_{(n)}$ will have no viable interpretation other than in terms of a probability estimate over the sets which were present in $\boldsymbol{S}$ but are not present in $\boldsymbol{S_C}$.

7.5 The mathematical function $\boldsymbol{M}$ cannot be derived in physically-real terms within $\boldsymbol{S}$.

If $\boldsymbol{M}$ could be deduced within $\boldsymbol{S}$ then it would be of the form of a recursive number-theoretic function over the $x_{(n)}$ that also holds for the real-number values of $x_{(r)}$. However, if $\boldsymbol{M}$ were deducible within $\boldsymbol{S}$, then its inverse $\boldsymbol{M}^{-1}$ would also be deducible within $\boldsymbol{S}$ and this inverse $\boldsymbol{M}^{-1}$ could then be used in $\boldsymbol{S_C}$ to replace the $x_{(r)}$ with the $x_{(n)}$ to give a modified scientific theory $\boldsymbol{S'}$ over natural-number valued variables. But Gödel's incompleteness theorems prove that any modified theory $\boldsymbol{S'}$ over the natural-numbers cannot be both consistent and complete. So $\boldsymbol{M}^{-1}$ cannot exist, which in turn means that $\boldsymbol{M}$ cannot be derived in physically-real terms within $\boldsymbol{S}$.

7.6 The theory will contain inconsistencies in any direct interpretation $\boldsymbol{I}$ of the theory $\boldsymbol{S_C}$, but these will not appear in any experimental predictions of the theory $\boldsymbol{S_C}$ given by the conversion function $\boldsymbol{M}$.

Similarly to 7.5, the simple interpretation $\boldsymbol{I}$ attempts to convert the complete theory $\boldsymbol{S_C}$ over the real-numbers $x_{(r)}$ into a complete theory over the natural-numbers $x_{(n)}$, but such a theory $\boldsymbol{S'}$ cannot be both consistent and complete. Since $\boldsymbol{S'}$ will have the same form as the complete theory and includes the undecidable property p, the natural-number theory $\boldsymbol{S'}$ resulting from the direct interpretation $\boldsymbol{I}$ of the complete theory $\boldsymbol{S_C}$ must be inconsistent.

8. Physical Systems

The postulates of *scientific materialism* given in section 2 form a common basis for scientific theories of the physical sciences, and the incompleteness proof of section 6 identifies a generic set of conditions in terms of a network of object reactions. Discrete objects were assumed to form the basis of material reality in 2.3, where the objects were assumed to be composed of fundamental objects in 2.4 and further assumed to participate in object conversion reactions in 2.8. These assumptions cover the occurrence of the object reaction networks of section 6 in a range of physical systems:

1) the chemical reaction network of the cellular metabolism (§8.2).
2) the organism 'reaction' network of an ecosystem (§8.3).
3) the socio-economic system of a nation (§8.4) where the production and consumption of traded-goods constitute the required 'object reactions'.

In all these cases, the incompleteness proof given in section 6 can apply within the scope of a physically-real scientific theory $\boldsymbol{S}$ of the system because the terms of $\boldsymbol{S}$ allow for object arithmetic and the construction of a potentially infinite network within the scope of the theory, despite the fact that the network in question is finite at all times (Goodband [2012a]).

The assumptions of fundamental objects 2.3 and object conversion reactions 2.8 also covers the case of particle reactions (§8.1), where the interconversion between mass and energy seen in particle reactions has already been included in the postulates of section 2 through the Relativity principles of 2.10. In this case, an infinite particle reaction network will be given in both a physically-real scientific theory $\boldsymbol{S}$ and reality if the assertion of a Vacuum Reservoir Effect (8.1.1) is true. Some sort of assertion of this form would be required in a physically-real scientific theory $\boldsymbol{S}$ of particles and radiation in order to account for the experimental measurements of the Casimir effect (Lamoreaux [1997], Bressi [2002]). The significance of considering the case of a physically-real scientific theory $\boldsymbol{S}$ based upon adding one system specific axiom (8.1.1) for particle reactions to the generic axioms of s*cientific materialism*, is that the theory $\boldsymbol{S}$ is proven to be incomplete. The application of the replacement procedure of section 7 to this incomplete physically-real scientific theory $\boldsymbol{S}$ of particle reactions provides the basis for the derivation of Quantum Theory (Goodband [2012a]).

8.1 Quantum Theory is the complete theory $\boldsymbol{S_C}$ of the form deduced in §7, which is required because the physically-real scientific theory $\boldsymbol{S}$ of particle reactions can be proven to be incomplete,

8.1.1 Vacuum Reservoir Effect: A physical system with energy E and particle number n can increase its energy ΔE and increase particle number N during some physical process of duration $\Delta t>0$, such that both the energy and particle number return to the values of E and n when the process has finished.

The Heisenberg Uncertainty Principle (Bransden [1989]) has essentially been converted into an assertion about the vacuum, where energy ΔE can be taken from the vacuum V and input into particle reactions, just as long as it is returned to the vacuum within time period $\Delta t>0$. The actual numerical value of the time-scale Δt is not relevant to the incompleteness proof, beyond it being non-zero. As any experimental measurement would be based upon the sort of physical process covered by the assertion, the assertion is metaphysical by its own terms. Within the time period Δt in which N extra particles exist, the vacuum state would be of the form V_N denoting the vacuum state as having a deficit of N particles.

8.1.2 Virtual-particle[2] reactions.

The existence of object reactions which interchange the types of the particle was assumed in 2.8, e.g. $A + B \rightarrow C + D$. Adding the metaphysical assumption of the Vacuum Reservoir Effect (8.1.1) to such real-particle reactions gives virtual-particle reactions which involve a change in the vacuum state, such as:

$$A + V_0 \rightarrow B + C + V_N$$

In this vacuum-particle reaction allowed by 8.1.1, the real-particle A reacts with the vacuum V_0 to produce virtual-particles B and C, where the vacuum state V_N carries the particle accounting required by 8.1.1 for the vacuum state V_0 to be restored when all the virtual-particles disappear, and we are just left with the real-particle A. For the purposes of the incompleteness proof of §6 the exact collection of particles and their particle reactions is not required. All that is needed is the existence of some finite set of particles which can interchange by particle reactions, and the Vacuum Reservoir Effect of 8.1.1.

8.1.3 At least one term γ denotes a wave.

This is not required for the incompleteness proof of §6, but is being assumed here so that it will be available for the view stated in 8.1.9

8.1.4 Particle arithmetic occurs because the vacuum state V_N acts as the control entity C_N of 5.2.

The vacuum state V_N gives the control condition required for a specific number of particles and particle reactions to be added to any particle state. This includes V_N being applied to a single particle state to give particle addition for some specific number N, and V_M being applied to such particle addition to give multiplication in particle numbers. Such a hierarchy could be repeated indefinitely by the terms of 8.1.1, but the implementation of addition and multiplication over particle numbers is sufficient for a realisation of the operations of arithmetic within the theory $\boldsymbol{S}$.

8.1.5 The vacuum-particle reaction network satisfies the conditions of the proof given in §6.

The Vacuum Reservoir Effect of 8.1.1 allows for the unlimited addition of virtual-particle reactions to any real-particle state, so as to give an unlimited network of particle reactions which includes arithmetic over the natural-number of virtual-particles (8.1.4). With 8.1.1 asserting that

2 Some label is required for the particle states that exist in conjunction with the vacuum state V_N, and virtual-particles is chosen because they will correspond to the virtual-particles of Quantum Theory.

such virtual-particles must be denoted in physically-real terms, the physically-real theory ***S*** must denote the full extent of the virtual-particle reaction network. This means that the arithmetic operations must be denoted within ***S*** (6.2), as well as the network growth process whereby additional reactions are added according to 8.1.1. So the function creation processes of substitution (6.4.2) and recursion (6.4.3) will be modelled within the scope of ***S***, and the unlimited nature of 8.1.1 means that the universality condition (6.4.4) will also be met. So the physically-real scientific theory ***S*** is proven to be incomplete. Note that the lack of a hidden variable theory for the complete theory $\boldsymbol{S_C}$ of particle reactions (i.e. quantum theory) is given by 6.7.2.

8.1.6 Every real-particle is given by a self-consistent virtual-particle reaction state, and so the sort of self-referential propositions which are undecidable in ***S*** could be describing the state of a real-particle. Assuming that this is true gives the prediction that every real-particle possesses at least one undecidable property.

This is experimentally verified by every particle possessing a wave property. The assertions of §2 give a classification distinction between particle motion and wave motion, such that it is not possible to start from physically-real terms denoting particles and deduce a wave property for an individual particle. This classification divide means that an observed wave property for a particle can definitively be known to be undecidable in the physically-real terms of ***S***.

8.1.7 The fields of Quantum Field Theory are continuous real-number valued descriptive fields for the numbers of particles.

The conversion process identified in §7 involves changing the natural-number valued variables $x_{(n)}$ of ***S*** denoting the numbers of particles with real-number valued variables $x_{(r)}$. The associated loss of the particle classification sets from ***S*** allows the undecidable wave property to be attached to these variables without causing a set-theoretic type conflict within the complete theory $\boldsymbol{S_C}$; note that there will still exist a set-theoretic type conflict for a direct interpretation ***I*** of $\boldsymbol{S_C}$ (8.1.8). These changes will give the variables $x_{(r)}$ the form of continuous real-number valued fields in space-time.

The Relativity principles of 2.10 mean that the descriptive field $x_{(r)}=\Phi$ for a single particle with its wave property will satisfy a Relativistic wave equation. For the scalar representation of the Poincaré symmetry group of Relativity, Φ will obey the Klein-Gordon wave equation (Ryder [1985]):

$$\partial^2\Phi + m^2\Phi = 0$$

Whereas for the fundamental spinor representation of the Poincaré group, the descriptive field $x_{(r)}=\Psi$ will necessarily obey the Dirac equation (Ryder [1985]):

$$\gamma^\mu\partial_\mu\Psi + m\Psi = 0$$

These simply follow from the principles of Relativity (2.10) which apply to any continuous field in space-time, and so will apply to the descriptive fields of a complete theory $\boldsymbol{S_C}$ of the form deduced in §7.

8.1.8 The mathematical function ***M*** of §7 is the origin of the process called the 'collapse of the wave-function' in Quantum Theory.

The inability to derive the conversion process ***M*** from the quantum fields to predictions is given by 7.5, and the existence of inconsistencies in the direct interpretation of quantum fields is given by 7.6, which are both characteristics of Quantum Theory. The complex character of the term Φ denoting the waves satisfying equations of 8.1.7 implies that the conversion process is of the form $\boldsymbol{x}_{(n)} = \boldsymbol{M}(\Phi^\dagger\Phi)$.

8.1.9 The descriptive fields for Relativistic spin of integer and half-integer eigenvalues are subject to the boson and fermion spin statistics.

The observable values $\boldsymbol{x}_{(n)} = \boldsymbol{M}(\Phi^\dagger\Phi)$ are subject to the conditions of Relativistic invariance (2.10) and material causation (2.8), which are sufficient conditions for the derivation of boson and fermion spin statistics for the descriptive fields Φ and Ψ (Mandl [1984]).

8.1.10 The same symmetry arguments used in particle physics for the introduction of electromagnetic fields etc. will also apply to the descriptive fields of $\boldsymbol{S_C}$ and so give descriptive field equations of the same form as those of Quantum Field Theory.

This origin for Quantum Field Theory was given for Quantum Electrodynamics in Goodband ([2012a]), and also discussed for Yang Mills theories.

8.1.11 Metaphysical interpretation of wave-particle duality.

The self-consistent particle reaction state of a particle p is conceptually given by the above as being of the form $p \to (p, \gamma) \to p$, where (p, γ) is some mixed combination of particle and wave states (8.1.3). The time scale of this transformation is less than the minimum time required for any experimental observation (8.1.1), and so every particle p will be observed to be in a dual state (p, γ) of particles and waves in any experiment.

8.2 The physically-real scientific theory $\boldsymbol{S}$ of the gene-protein system of a living cell is incomplete.

8.2.1 All the chemical molecules inside a cell are the objects being classified into sets by type (2.3), where the numbers of chemicals in these sets is changed by the chemical reactions that occur between the different chemicals (2.8).

8.2.2 The control entity C_N required for physically-real object arithmetic (5.2) could be realised in terms of a sequential molecule which has a property that changes at some number N, or in terms of a sharp phase-transition in chemical state due to antagonistic chemical reactions.

For the case of a sequential molecule C_n of n chemical units, the molecule C_n would promote some other chemical reaction which realises the successor function $s(n)$ for some chemical type for $n<N$. When the control molecule contains N units it would be required to change its properties such that it no longer promoted the other chemical reaction. In this way it could increase the number of some other type of chemical molecule by N units, i.e. the addition of $+N$.

For the phase-transition case, the net result of antagonistic chemical reactions could suddenly jump when the number of molecules of some chemical type exceeded N. The control entity C_n in this case is a effectively a set of molecules of some chemical type which are incrementally increased in number. When the number reaches N, the phase-transition characteristic would then be required to inhibit the chemical reaction producing the chemical molecules in question, and so realise a net increase of $+N$ in the numbers of such chemical molecules.

Repeats of these control mechanisms over chemical processes which realise object addition would then realise multiplication in object numbers, and in this way the operations of arithmetic would be present in the chemical system. This would have to be expressed in the scientific theory $\boldsymbol{S}$ in physically-real terms, and so the operations of arithmetic would be realised in physically-real terms within the scope of the theory $\boldsymbol{S}$ itself.

8.2.3 Metabolic network of chemical sequences and cycles.

The individual chemical reactions of a living cell are linked up to form chemical sequences and cycles, which are linked together through the common genetic system of protein production, and the common energy molecules of ATP and NADH, to form a metabolic network. The modelling of this network in the scientific theory $\boldsymbol{S}$ will give a realisation of function substitution (6.4.2) within the scope of the theory $\boldsymbol{S}$.

8.2.4 Random genetic mutations are a source of ongoing variation in the network structure.

Mutations of the genetic templates of proteins will give rise to new variations of protein, some of which will catalyse new chemical reactions that produce new types of chemical molecule within the metabolism of a living cell. Although such mutations are random and so not directly caused by any component of a cell, they are nonetheless the result of protein actions. So any attempt at a complete scientific theory $\boldsymbol{S}$ in strictly physically-real terms would need to also describe the occurrence of random variation within the metabolism of the cell. This would give a realisation of function recursion (6.4.3) within the scope of the theory $\boldsymbol{S}$.

8.2.5 The scientific theory $\boldsymbol{S}$ of the gene-protein system can satisfy the conditions of §6.

The random variation in the genetic templates of proteins can occur at all times and so will occur on an ongoing basis. This provides a realisation of the universality condition (6.4.4) within the scope of the theory $\boldsymbol{S}$. It must be noted that there is a clear distinction between the occurrence of an infinite network in reality, which is what assumption 8.1.1 leads to for particle-reactions, and the ability to construct an infinite network within the scope of a scientific theory $\boldsymbol{S}$ in strictly physically-real terms, which is what can occur in this case. The physical system itself is finite at all times, and any undecidable propositions about a causally closed or self-consistent dynamic state of the metabolic network will also be finite. It is the dependence of the proof upon the universality condition (6.4.4) which requires an infinite network, but the proof only requires an infinite network to be realised in physically-real terms within the scope of the theory $\boldsymbol{S}$, and not in reality. So the scientific theory $\boldsymbol{S}$ itself is proven to be incomplete under these conditions.

8.3 The physically-real scientific theory $\boldsymbol{S}$ of an ecosystem is incomplete.

8.3.1 All the organisms in an ecosystem are objects being classified into sets by type (2.3), where predation and reproduction change the numbers of organisms in these sets (2.8).

8.3.2 The control entity C_N required for physically-real object arithmetic (5.2) can be given by behavioural changes in organisms after a certain numbers of object transformation behaviours have been repeated.

The object transformation behaviours in question can be given by eating another organism, or by the symbiotic exchange of chemicals. Such feeding behaviour can trigger a change of behaviour when it has been repeated some number of times, of which an important example is reproducing after having accumulated a sufficient reserve. This is perhaps most obvious in uni-cellular organisms which divide (realising the +1 of the successor function $s(n)$) when they have reached a particular size.

8.3.3 The full food web of an ecosystem gives an interconnected network of organism predation and symbiotic exchange.

The ecosystem network described by the physically-real scientific theory $\boldsymbol{S}$ will only fully model the ecological reality when it describes all the organisms in the ecosystem, including all the species of fungi and bacteria which can reproduce. Note that such full accounting would also have to model the numbers of mitochondria and chloroplasts inside the cells of organisms.

8.3.4 Random variation gives unending variation in the description of the network structure.

As for 8.2.4, random variation of the network adds a physically-real realisation of function recursion (6.4.3) to the network realisation of function substitution (6.4.2).

8.3.5 The scientific theory $\boldsymbol{S}$ of an ecosystem can satisfy the conditions of §6.

As for 8.2.5, unending random variation in the organisms of an ecosystem gives a realisation of the universality condition (6.4.4) and means that the incompleteness proof can be constructed in strictly physically-real terms within the scope of the scientific theory $\boldsymbol{S}$. The ecosystem itself remains finite at all times, and any undecidable propositions about a causally closed or self-consistent dynamic state of the ecosystem will also be finite.

8.4 The physically-real scientific theory $\boldsymbol{S}$ of the socio-economic system of a nation is incomplete.

8.4.1 All the people and traded-goods being exchanged in a socio-economic system are the objects being classified into sets by type (2.3), where manufacturing and consumption change the number of goods present in these sets (2.8).

8.4.2 The control entity C_N required for physically-real object arithmetic (5.2) can be realised as a behavioural change after some number N of goods have either been produced or consumed.

Such behavioural changes give a means for the realisation of arithmetic over the numbers of traded-goods, and also over the numbers of individuals in each of the occupations within the socio-economic system.

8.4.3 The socio-economic system of a nation forms a network which will tend to occupy dynamic self-consistent states.

All the different supply chains for traded-goods link together to form a network because of connections between common suppliers, distributors and consumers. This will give a network realisation of function substitution (6.4.2) in the scientific theory ***S***. When the traded-goods of the socio-economic system are only made in order to be sold to consumers, the economic costs of production will act to force the number of produced goods to match the number of goods bought. As this condition will apply to each and every traded-good of the socio-economic system, this will tend to drive the socio-economic system into self-consistent states where sellers match buyers.

8.4.4 Innovation is a source of potentially unending variation in the traded-goods and occupations of a socio-economic system.

Although innovation is not technically random, it can satisfy the same role as played by purely random variation in 8.3.4 if it is always possible for innovation to create a new traded-good or occupation that has not existed before. When this is the case, the modelling of innovation in the scientific theory ***S*** will give a realisation of both function recursion (6.4.3) and the universality condition (6.4.4) in physically-real terms within the scope of the theory ***S***.

8.4.5 The scientific theory ***S*** of an ecosystem can satisfy the conditions of §6.

As for 8.3.5, the theory ***S*** of a finite socio-economic system meeting these conditions could possess a self-consistent dynamic state which is described by a finite undecidable proposition.

9. Paradigm Shift

The view of a scientific theory ***S*** used in the course of this paper is one where ***S*** has an axiomatic basis P_A in which the axioms can be divided into the general set of section 2 and a system specific set. This division gives a basis for the classification of *paradigm shifts* where there is a change in the set of axioms P_A: the change is in the general set; or the change is in the system specific set. Such *paradigm shifts* just within individual scientific disciplines will tend to just involve changes to the system specific axioms, as changes to the general set of axioms will change the axiomatic basis for all of science. Historically, there have been two paradigm shifts where it would appear that the fundamental axioms of science have been subject to change: the scientific revolution of Copernicus and Galileo; and the introduction of Quantum Theory in physics.

The scientific revolution of Galilean mechanics was associated with the metaphysics of Aristotle being replaced by a new metaphysical basis for the study of mechanical bodies, and some of the metaphysical assertions given in section 2 can historically be traced back to Galileo. Equally significantly for the purposes of scientific theories, Galileo's use of mathematical terms to describe the physical world marked the beginning of the correspondence rules for physically-real terms (§3). So in the terms used in this paper, the scientific revolution of Galilean mechanics was associated with changes to the axiomatic basis of science, as for the restricted definition of a *paradigm shift* given earlier, but also involved a change in the descriptive framework.

The historical view of the creation of Quantum Theory was that of a *paradigm shift* involving very significant changes to the metaphysical basis of classical physics (§2), but the derivation of Quantum Theory in section 8 indicates that this was erroneous. The ability to derive Quantum Theory (§8.1) from the incompleteness proof of section 6, and its descriptive solution of section 7, without changing any of the axioms of section 2, implies that the paradigm shift of Quantum Theory was actually about a change in the correspondence rules used for scientific notation. This gives a very different form of paradigm shift, one not due to a discontinuous jump in axiomatic basis for the scientific theory of some physical system, but one entirely due to a notational shift. Such a change in the descriptive framework has an effect on what can be said, and what questions can be addressed, in much the same way that changes to the underlying metaphysical assumptions do. This identification of a different class of scientific paradigm shift has a relevance beyond Quantum Theory, because the other incompleteness proofs of section 8 appear to indicate that Quantum Theory was just the first instance of a more widespread paradigm shift in science that is yet to come. This section outlines the character of this class of paradigm shift due to changing the basis of notation used in the construction of scientific theories.

9.1 There-exists a formal language $\boldsymbol{L}$ (§1) suitable for denoting the reality of §2 in physically-real terms (§3), such that a set of axioms P_A, true statements P and deducible statements P_D can be made about any physical system in reality.

9.2 *Materialism*: the set of all true statements P expressed in the physically-real terms of §3 in formal language $\boldsymbol{L}$ constitute all the true statements that can be said about reality.

9.3 *Mechanistic paradigm*: the set of statements P_D that can be derived from the set of axioms P_A within the scope of a scientific theory $\boldsymbol{S}$ in strictly physically-real terms constitute all the true statements of reality P expressible in the formal language $\boldsymbol{L}$, i.e. it is asserted that $P_{D.} \equiv P$.

9.4 Scientific incompleteness: there-exists a non-empty subset P_U of P where $P_U \cap P_{D.} = \{\}$.

The claim that the *mechanistic paradigm* (P_D) accounts for the reality of *materialism* (P) is proven to be incorrect in §6 by proving $P_U \neq \{\}$, and this is verified in §8.1.

9.5 *Dualistic paradigm*: the physically-real terms of $\boldsymbol{S}$ are replaced with non-physically-real terms that incorporate the observed undecidable properties[3] to give a complete theory $\boldsymbol{S_C}$ (§7). A mathematical conversion function $\boldsymbol{M}$ also has to be added in order to obtain predictions for the physically-real terms of experimental measurement.

The character of $\boldsymbol{M}$ is such that it will inevitably involve a probabilistic interpretation of the predictions made by the complete theory $\boldsymbol{S_C}$.

9.6 Paradigm shift in science from the *mechanistic paradigm* (9.2) to the *dualistic paradigm* (9.4).

The paradigm shift involves a switch from the physically-real scientific theories of §4 based upon using physically-real terms (§3) to the use of non-physically-real terms in scientific theories (§7). This will only be necessary for those physical systems for which physically-real scientific theories can be proven to be incomplete. When this is not the case, physically-real scientific theories constructed within the *mechanistic paradigm* will be complete, such that every proposition p about observations of the physical system will be derivable in physically-real terms, i.e. $p \in P_D \; \forall p \in P_O$.

3 They have to come from observation because they cannot be derived

10. Reification

The paradigm shift of section 9 from the *mechanistic paradigm* (9.2) to the *dualistic paradigm* (9.4) is given in terms of a shift in the notation used in scientific theories. However, discussions of the observations of reality are generally conducted in natural-languages, including much of the discussion of experiments in science, and the experience of Quantum Theory indicates that the paradigm shift of section 9 induces a change in the use of natural-language. The incompleteness proofs of section 8 beyond that of the particle reactions would seem to indicate that the more widespread paradigm shift discussed in section 9 will lead to problems in using natural-language to describe these systems.

The defining character of the shift to 'another way' of describing the physical systems of section 8 is revealed by adopting a linguistic view of denoting the countable number of objects as if their numbers could take the values of the continuous real-numbers: 'as if' is the linguistic construction of a metaphor. Since the construction of scientific theories expressed in the deductive language of mathematics has had to resort to the use of linguistic metaphor, this would imply that natural-language descriptions are also going to have to involve the use of metaphor. However, the use of metaphor within the context of the *scientific materialism* of section 2 would seem to be vulnerable to the reification of such metaphors as being real fields or 'things' of some form. This can be seen in the history of the discussion of Quantum Theory in natural-language within the context of science. In wider society, such a reification of the field terms of Quantum Theory resulted in the appearance of mystical views where the reified 'quantum fields' were taken to give a basis for a new form of vitalism. The adoption of such mystical views will be framed in terms of the 'category mistake' of Ryle ([1949]), where such reifications will be given Ryle's label of being 'ghosts'. However, the universal assertion of the *mechanistic paradigm* (9.3) also leads to a complementary category mistake where every physical system is classified as being 'machine'. Such a view will be given the label of *machinism*, to complement the mystical views of *vitalism* that the reifications of metaphorical description are real 'things' of some form. The initial preference of 'ism' for the mechanistic view would be the term mechanism, but this term is used with two different meanings. The first usage is the assertion that the simple mechanical actions of objects underlie all the observations of material reality, which is the same as what was labelled as *materialism* in section 9. The second usage of mechanism is that every observation can be derived in mechanical terms, which was labelled *machinism* in section 9. The issue identified in this paper is that these two meanings are different, and there is a gap between *machinism* and the full domain of *materialism* addressed by science. So the designation of *machinism* is being used in order to handle this difference, because the meaning of mechanism is too ambiguous.

The philosophical framework given here for addressing the views of mysticism give some final points of comparison with Wittgenstein's *Tractatus* ([1921]), and include a series of propositions nakedly asserting that the reification of the undecidable features of physical systems like those in section 8 forms the observational basis for the views of vitalism in all its forms, and this is why such mystical views persist despite the fact that they involve 'ghosts'. The metaphysics of the *Tractatus* is different from the *scientific materialism* given in section 2, and this has resulted in significant differences in conclusion despite this paper sharing the theme of the use and limitations of language in the description of reality. This paper has been very focused on the use of formal language for deductive purposes, and has only considered the issue of what constitutes scientific knowledge in so far as to recognise a distinction between metaphysical knowledge and observational knowledge. The benefit of this descriptive focus has been to identify the limitations of a particular form of scientific language, such that these limitations can be overcome by changing the form of scientific language used.

10.1 *Machine* category: the set of propositions P_D about the components of a physical system that can be derived within the scope of a physically-real scientific theory $\boldsymbol{S}$ in formal system $\boldsymbol{L}$ from the set of axioms P_A.

This corresponds to the domain of the *mechanistic paradigm* of 9.3, and the assertion of the universality of the *mechanistic paradigm* is taken to define *machinism*, i.e. the claim that the set P_D is the same as the set of all true propositions P that can be made about the physical system in reality.

10.2 *Ghost* category: the collection of observed undecidable features P_U which are expressed in the formal language $\boldsymbol{L}$ ($P_U \subset P$), but cannot be derived ($P_U \cap P_D = \{\}$).

The assertion that the features of P_U are due to the existence of some undiscovered component of reality is taken to define *vitalism*.

10.3 *Mechanistic duality*: the set of observable propositions P is given by the pair (P_D, P_U) where the set P_D can be derived within some formal system $\boldsymbol{L}$, and the set P_U are described within some descriptive system $\boldsymbol{M}$ that is independent of $\boldsymbol{L}$, to give a dual philosophical system ($\boldsymbol{L}$, $\boldsymbol{M}$).

No further assumptions will be made about the character of $\boldsymbol{M}$, but it can be noted that it is unlikely to be a logical deductive system and it cannot share the same basis as $\boldsymbol{L}$. As *materialism* has been defined to be described by the full set of propositions P (9.2), a complete description of reality in physically-real terms will split into the pair (P_D, P_U) where the view of *materialism* given by *mechanistic duality* is given by (*machinism*, *vitalism*). This can be expressed in terms of the existence of a gap between *materialism* (9.2) and *machinism* (10.1) which is filled by *vitalism* (10.2), i.e. philosophically, *materialism* – *machinism* = *vitalism* ≠ {}.

10.4 Apparent inference of top-down causation from the system level domain of $\boldsymbol{M}$ to the component level domain of $\boldsymbol{L}$.

The undecidable propositions of P_U are statements in physically-real terms about causally closed arithmetic states of an object system (this is based on the conditions of §6 apparently being verified in §8.1), where the propositions cannot be derived from statements about the object content of the system. In the dual descriptive system ($\boldsymbol{L}$, $\boldsymbol{M}$), causal closure would be expected to involve the descriptions of $\boldsymbol{L}$ giving rise to those of $\boldsymbol{M}$, which then in turn give rise to those of $\boldsymbol{L}$, i.e. the closed cycle $\boldsymbol{L}\rightarrow\boldsymbol{M}\rightarrow\boldsymbol{L}$. However, the undecidable propositions at the system level described in $\boldsymbol{M}$ cannot be deduced from the component level descriptions of $\boldsymbol{L}$, which can be taken to imply that the step $\boldsymbol{L}\rightarrow\boldsymbol{M}$ doesn't exist. This would just leave the second part $\boldsymbol{M}\rightarrow\boldsymbol{L}$, which can then be interpreted as top-down causation from the system level description of $\boldsymbol{M}$ to the component level description of $\boldsymbol{L}$.

10.5 *Ghost* in the m*achine*: the interpretation of (P_D, P_U) in the dual philosophical system ($\boldsymbol{L}$, $\boldsymbol{M}$) in terms of top-down causation from $\boldsymbol{M}$ to $\boldsymbol{L}$ is accompanied by the reification of one or more of the elements of $\boldsymbol{M}$ as being real 'things' in reality.

Such reifications are 'ghosts' in the sense of 'appearing but not actually existing' and are due to the categorisation mistake of mistaking the elements of the g*host* category (10.2) as being real physical 'things'. In reality, these 'ghosts' are just figments of a mistaken philosophy.

10.6 *Quantum myth* (§8.1): the reification of the descriptive fields of Quantum Theory as being real physical fields that affect top-down causation of the form $\boldsymbol{M}\rightarrow\boldsymbol{L}$.

In biology, a reified 'quantum field' has been viewed as being the 'thing' that animates the chemical components of a cell to bring it to life, or the 'shaping field' that determines the final form of a multi-cellular organism. In physics, such reification of the descriptive 'quantum fields' of §8.1 as being real physical fields underlies the assumption that Quantum Theory is fundamental in physics, such that gravity also has to be similarly 'quantised'. The points given here indicate that such reified 'quantum fields' are 'ghosts', just figments of a mistaken philosophy. This would explain why all attempts at physics unification based upon this assumption have failed (Goodband [2012a, 2012b] outlines a more successful approach).

10.7 Proposition of Life (§8.2): the state of Life is a self-consistent arithmetic state of the gene-protein system of a living cell which possesses one or more undecidable features.

The universality required for the incompleteness proof is achieved through the inclusion of ongoing random genetic mutations in the scientific theory $\boldsymbol{S}$, which means that the incompleteness proof is effectively constructed over an infinite evolutionary time-scale within the theory ***S***. This proposition is dependent upon a finite statement about a finite system being undecidable, which is shown to be possible by the halting problem where a finite Turing Machine encounters a finite undecidable statement. The same condition also underlies all the remaining propositions.

10.8 Proposition of Form: the final developmental form of a multi-cellular organism possesses one or more undecidable features.

The basis for this proposition does not lie with the proofs of §6 and §8 but with a cellular realisation of a Turing Machine in physically-real terms (Goodband [2012a]). The proposition is simply given here as an unjustified assertion because the issue of a 'shaping field' naturally arises in this section.

10.9 Proposition Mind: the psychological state of the brain of a highly social mammal can possess one or more undecidable features.

The network-based incompleteness proofs of §6 and §8 do not apply directly to the network of neurons because neurons only come in a finite number of types, which means that a neural network cannot directly satisfy the universality condition 6.4.4. However, the formation of a network of memories can satisfy the conditions of §6 under special circumstances (Goodband [2012a]), but whether these circumstances are realised in the human mind is a different matter. The proposition is given here as an unjustified assertion because of the history of Gödel's incompleteness theorems in the context in the mind, but there are two points to note about the Mind result of Goodband ([2012a]): it is about memory linkage, not about rational reasoning; and an undecidable feature is undecidable in theory, which means that the feature cannot be predicted to be consciousness.

10.10 Proposition of Nature (§8.3): the state of Nature is a self-consistent arithmetic state of the ecosystem formed by the network of interacting organisms which possesses one or more undecidable features.

As for Life (10.7), the incompleteness proof is effectively constructed over an infinite evolutionary time-scale in theory, because the universality required comes from unending random variation.

10.11 Proposition of the Market (§8.4): the state of the Market is a self-consistent arithmetic state of the socio-economic system of a nation which possesses one or more undecidable features.

It must be noted that the required arithmetic can just be in terms of the traded-goods only, the use of money is not specifically required for the incompleteness proof. On the other hand, the self-consistency associated with causal closure is most obviously achieved by the use of money in the socio-economic system.

10.12 Proposition of Us (§8.4): the state of Us is a self-consistent arithmetic state of the social system of a society which possesses one or more undecidable features.

As money is implicitly of the form of an I.O.U.[4] the same enforcement of causal closure realised by the use of money could also be realised in a society that didn't use money, but instead implemented and maintained strict I.O.U. reciprocity in terms of the value of goods and services exchanged. The incompleteness proof of §8.4 would then still stand for such a system, despite of the lack of money. This state is given the social label of Us because it is possible that it could exist independently of the economic Market state of 10.11.

10.13 Proposition of Religion: a religion asserts top-down causation $\boldsymbol{M} \rightarrow \boldsymbol{L}$.

This can be accompanied by the assertion that the domain of the religious system of description ***M*** is about a different domain of reality from the domain of the physical world described by ***L***.

10.14 Proposition of Religious Reification: God is Us.

This proposition is based upon the reification of the collective societal state of Us (10.12) and the religious assertion of top-down causation (10.13)

4 The bank notes of some societies explicitly take the form of the Central Bank saying I.O.U.

References

Bransden, B.H. and Joachain, C.J. [1989], *Introduction to Quantum Mechanics*, Longman Scientific & Technical (Harlow)

Bressi, G and Carugno, G and Onofrio, R and Rusos, G [2002]: *Measurement of the Casimir Force between Parallel Metallic Surfaces*, Physical Review Letters 88, 4

Feyeraband, P [1975]: *Against Method*, London: Verso

Gödel, K [1931]: *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, Dover Publications, 1992

Goodband, M. J. [2012a]: *Agent Physics: On the Incompleteness of Physically-Real Scientific Theories*, 2012, ISBN 978-0-9571490-0-7

Goodband, M.J. [2012b], *S^{10} Unified Field Theory*, (http://mjgoodband.co.uk/papers/s10-unified-field-theory.pdf)

Kuhn, T [1962]: *The Structure of Scientific Revolutions*, 2nd. ed., Chicago: Univ. of Chicago Pr., 1970 ISBN 0-226-45804-0

Lamoreaux, S.K. [1997]: *Demonstration of the Casimir Force on the 0.6 to 6μm Range*, Physics Review Letters 78, 5

Mandl, L and Shaw, G [1984], *Quantum Field Theory*, John Wiley & Sons

Mendelson, E [2010], *Introduction to Mathematical Logic* (Fifth Edition), CRC Press, Taylor & Francis Group (Boca Raton)

Popper, K [1935]: *The Logic of Scientific Discovery*, Routledge Classics, 1992 ISBN 978-0-415-27844-7

Russell, B and Whitehead, A [1910-13]: *Principia Mathematica* , Cambridge: Cambridge University Press

Ryder, L [1985], *Quantum Field Theory*, Cambridge: Cambridge University Press

Ryle, G [1949]: *The Concept of Mind,* Penguin Classics, 2000 ISBN 978-0-14-118217-9

van Fraassen [1970]: *On the extension of Beth's semantics of physical theories*, Philosophy of Science, 37, 325-39

Wittgenstein, L [1921]: *Tractatus Logico-Philosophicus*, Routledge Classics, 2001 ISBN 978-0-415-25408-3