S^{10} Unified Field Theory

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Abstract: The Incompleteness results of Agent Physics [1] showed that a Quantum Field Theory is the necessary form of a complete scientific theory of particle-reactions, implying that unification of the 4 forces of Nature should be sought at the classical level. A class of Classical Unified Field Theory in 11-dimensions was presented in Agent Physics as a pure metric field theory, i.e. without additional fields, where a wormhole configuration produces a topological transition from a unified spatial 10-sphere S^{10} to the spatial product space $S^3 \times S^7$. The initial transition is followed by inflation of the S^3 cosmology and compactification of the $\underline{S}^7$ particle dimensions. The $\underline{S}^3$ fibre of the $\underline{S}^7$ fibre bundle is identified with the colour symmetry and the $\underline{S}^4$ base-space is identified with the Electroweak symmetry. A non-trivial Electroweak Vacuum mapping from the Electroweak base-space $\underline{S}^4$ to the S^3 cosmology beaks the symmetry of the $\underline{S}^7$ fibre bundle, with the Electroweak symmetry breaking $\underline{S}^4 \rightarrow \underline{S}^3 \times \underline{S}^1$ giving rise to topological monopoles. The eigenvalues of the symmetry groups of the quotient space $SU(4)/SU(3) \cong \underline{S}^7$ yields a 3 by 4 table of group eigenvalues for the topological monopoles, which are identified with the spectrum of fermions. The classical theory of the topological monopoles has local symmetry group $SO(3)_C \otimes SU(2)_I \otimes U(1)_Y$, where the global "symmetry group" of the expected Grand Unified Theory (GUT) of particle symmetries is given by $SU(4)/SU(3) \cong S^7$. The classical theory of the monopoles is mathematically incomplete and the required form of the complete theory is that of a Quantum Field Theory. This review paper presents a short derivation and discussion of the S^{10} Unified Field Theory (STUFT) class of metric field theory.

Any physical causal space in D spatial dimensions will have a causal metric $g_{\mu\upsilon}$ which is locally flat. A metric space is a topological space on which a metric is defined, where a metric is a quadratic form over the space, which is in turn a form of normed division algebra, and only 4 normed division algebras can be defined over the real numbers:

- Real numbers: dimension 1
- Complex numbers: dimension 2
- Quaternions: dimension 4
- Octonions: dimension 8

A physical event A can only be the physical cause of physical event B if their separation satisfies:

$$ds_D^2 = g_{\mu\nu} x^\mu x^\nu = -c^2 t^2 + \sum_{i=1}^{D} x_i x_i \leq 0$$

where the limit $ds_D = 0$ is given by metric wave radiation travelling at speed c. Whatever the actual form of the metric field equations for the metric space, their solution for the value of some vector x^μ determined by causal events not due to metric waves, will inevitably involve taking the square-root of the separation:

$$ds = \sqrt{-1}\,|ds| \qquad \text{for} \qquad ds^2 = g_{\mu\nu} x^\mu x^\nu < 0$$

So physical causal space will be taken as being defined by the following assertion:

- physical causation will only be consistent and complete if the physical causal manifold realises **all** the closed homogenous and isotropic manifolds in the spaces definable from the square-roots of –1

This identifies the following unit spheres in the spaces of the corresponding normed division algebras:

- S^0 manifold of {–1,+1}
- S^1 manifold of the unit complex numbers
- S^3 manifold of the unit quaternions
- S^7 manifold of the unit octonions

The product space of the continuous manifolds $S^1 \times S^3 \times S^7$ is that of a cyclical $\cancel{S}^1$ (strikethrough will be used to distinguish the time dimension from the spatial dimensions) closed spatial cosmology S^3 with particle space S^7 at all points in 3+1 dimensional space-time. The discrete manifold S^0 corresponds to a space of monopoles (+1) and anti-monopoles (–1), which requires an appropriate topological transition to yield them as topological monopoles.

Imposing the condition of unification on the product space $S^3 \times S^7$, in addition to the homogeneity and isotropy of spheres, gives the unification space as being S^{10}. The topology of a sphere S^n can be transformed to the product space of a torus $T^{n+m} = S^n \times S^m$ by the insertion of a hole through the sphere, which would topologically be of the form of a Rosen-Einstein bridge or wormhole solution in a geometric theory of the form of an extension to General Relativity. Now the formation such an internal bridge by incoming depressions will be homotopic to two hemispherical depressions in the sphere meeting and fusing at the equator of the hemispheres. As this joining is equivalent to the equatorial map, the formation of such internal bridges can result in non-trivial windings as $\pi_n(S^{n-1}) = Z_2\ \forall n>2$, which would result in the torus T^{n+m} also having a twist in it, where $\pi_m(S^n) = Z_2$ is required for consistency. For S^{10} there are 3 possibilities for the topological transition which are compatible with the formation of a $\pm\pi$ twist:

$S^{10} \rightarrow T^{2+8} = S^2\times S^8$ as $\pi_8(S^2) = Z_2$
$S^{10} \rightarrow T^{3+7} = S^3\times S^7$ as $\pi_7(S^3) = Z_2$
$S^{10} \rightarrow T^{4+6} = S^4\times S^6$ as $\pi_6(S^4) = Z_2$

The first space S^n would be that of the spatial cosmology of the unified space, and S^m would be the particle space. The second map corresponds to the case of a cosmology with 3 spatial dimensions, and a particle space of S^7.

Now S^7 is a Hopf fibre-bundle of an S^3 fibre over an S^4 base-space and the non-trivial mapping of the homotopy group $\pi_7(S^3)=Z_2$ is due to the equatorial map from the S^4 base-space to the S^3 cosmology, $\pi_4(S^3)=Z_2$. As the equatorial map involves mapping the S^3 equator of S^4 to the spatial S^3 cosmology, such a non-trivial map would break the equivalence of the 7 particle dimensions to give a sequence locally of the form:

$$S^{10} \rightarrow S^3\times S^7 \rightarrow S^3\times(S^3\times S^4) \rightarrow S^3\times(S^3\times(S^3\times S^1))$$

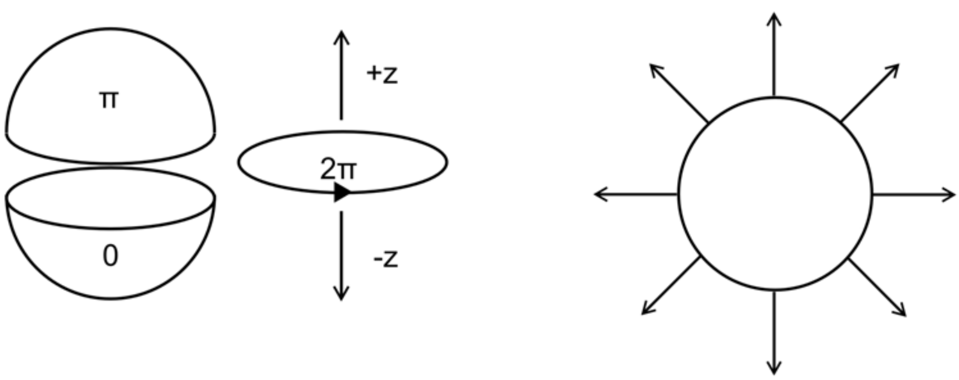
The first transition is the topological transition given by the formation of a wormhole solution in S^{10}, the second is the splitting of the S^3 fibre from the S^4 base-space needed for the non-trivial map of the S^3 equator of S^4 around the spatial S^3 cosmology. Given this pattern, the unified particle space S^7 will be described as being the S^3 colour-fibre over the S^4 electroweak base-pace, and so the non-trivial winding around the S^3 cosmology is that of the Electroweak Vacuum.

Topological Monopoles

The equatorial map involves dividing the equatorial sphere S^{n-1} into "northern" and "southern" hemispheres which are then rotated by 2π with respect to each other around the residual S^1 in the full S^n. Figure 1a shows the equatorial map for the mapping of S^3 to S^2, corresponding to the symmetry breaking SU(2) → U(1) in a field theory, as the group space of SU(2) is S^3. Under the application of a 2π rotation the SU(2) generator σ_z transforms as $\sigma_z \rightarrow -\sigma_z$, and so the equatorial map gives the σ_z generator pointing in the same direction everywhere.

In Relativity, the rotation group generators are given by the Pauli matrices $J_i=\sigma_i/2$ of SU(2) and so if there existed a spherical hole in space, the equatorial map could also be constructed in terms of the rotation group. The rotation generator J_z would be aligned upwards with the z-axis in the northern hemisphere, and aligned downwards against the z-axis in the southern hemisphere, with the two hemispheres being joined at the equator by the 2π rotation of an S^3 great circle, which would reverse the alignment of the rotation group generator $J_z \rightarrow -J_z$. By the symmetry of the sphere S^2, this equatorial map could exist for all orientations of the x, y and z-axes. As this configuration requires a 4π spatial rotation for rotational invariance, it will have a rotation group eigenvalue of ½, which means that under the full Poincaré group of Relativity it will transform as a spin ½ object.

Figure 1: a) Equatorial map b) Fibre-bundle hedgehog



This equatorial map also underlies the construction of a Dirac magnetic monopole, as the U(1) gauge field configuration can be expressed as:

$$A_\phi^N = \frac{q_m}{r\sin\theta}(1-\cos\theta)(I_Q\hat{z}) \qquad A_\phi^S = \frac{q_m}{r\sin\theta}(1-\cos\theta)(-I_Q\hat{z}) \qquad \Phi = \oint A.dS = 4\pi\, q_m$$

where I_Q denotes the generator of the electric U(1) symmetry group and the magnetic permeability μ has been absorbed into the magnetic charge q_m.

The symmetry breaking SU(2) → U(1) in a field theory is also subject to the general homotopy group relation:

$$\pi_2(G/H) = \pi_1(H) \qquad \text{with } \pi_1(U(1)) = Z$$

which says that there-exist topological monopoles of the classical "hedgehog" configuration shown in Figure 1b. This second homotopy group relation arises for SU(2) → U(1) because S^3 is also a Hopf fibre-bundle of an S^1 fibre over an S^2 base-space, and there-exist non-trivial maps from the S^2 base-space to the spatial sphere S^2. The general homotopy relation also applies to any sequence of the form $S^n \to S^m \times \ldots \times S^1$, and so the topological transition in S^{10} will give rise to topological monopoles. So unlike Kaluza-Klein Theory, which unifies the forces of gravity and electromagnetism but contains no particles, the corresponding geometric theory containing the transition $S^{10} \to S^3 \times S^7$ and a non-trivial Electroweak Vacuum will contain monopole topological defects. Whereas the equatorial map of S^3 to S^2 will give magnetic monopoles, the fibre-bundle map will give electric monopoles of the form:

$$A_0 = \frac{q_e}{4\pi\, r} I_Q \hat{r} \qquad E_r = \frac{q_e}{4\pi\, r^2} \qquad \Phi = \oint E.dS = q_e$$

where the electric permittivity ε has been absorbed into the electric charge q_e. The geometric theory would be expected to contain both electric and magnetic charges – and so possess electromagnetic duality – because both types of non-trivial mapping exist for the S^3 fibre-bundle. This topological basis for the electric and magnetic monopoles gives the Dirac quantisation condition $q_e q_m = \frac{1}{2} n$ (in dimensionless units) for the U(1) gauge transformation joining the two hemispheres of the Dirac monopole configuration at the equator:

$$A_\phi^S = A_\phi^N - \frac{2q_m}{r\sin\theta} = A_\phi^N - \frac{i}{q_e}\lambda \nabla_\phi \lambda^{-1} \qquad \lambda = \exp(2iq_e q_m \phi)$$

Such topological monopoles will arise in the S^{10} geometric theory because of the non-trivial Electroweak Vacuum, but the irrational value of the Weinberg angle would seem to imply that topological monopoles do not exist in the Standard Model. However, a distinction must be drawn between the group Weinberg angle φ_W giving the symmetry embedding of the U(1) of electromagnetism, and the physical Weinberg angle θ_W giving the relationship between the dimensions of the particle space. In the Electroweak Vacuum map, the S^3 equatorial sphere corresponds to the space of isospin, and the remaining S^1 corresponds to the space of hypercharge. The 2π rotation at the equator reverses the isospin alignment of the "northern" and "southern" hemispheres of the configuration such that the isospin orientation is the same throughout space. This means that the isospin eigenvalue of the Electroweak Vacuum is ½ and the symmetry group of the S^3 isospin space is SU(2), as opposed to SO(3) with integer group eigenvalues. The 2π hypercharge rotation at the equator gives an eigenvalue of 1 for the U(1) hypercharge symmetry, and so the group Weinberg angle is $\tan\varphi_W = \frac{1}{2}$ – a rational value for which topological monopoles exist.

Now the surface of any sphere S^n can be defined to be the set X of coordinate tuples $(x_0, \ldots, x_n)$ in (n+1)-dimensions, which satisfies the invariant:

$$r_n^2 = \sum_{i=0}^{n} x_i^2$$

For any sphere S^n, the cardinality (n+1) of the tuple $(x_0, \ldots, x_n)$ is invariant for all values of the radius r_n. So the set X defining a particular sphere S^n will be uniquely characterised by the tuple $(r_n, n+1)$, such that two different spheres will only be equivalent if their $(r_n, n+1)$ tuples are the same. For the sphere-to-sphere map of S^n to S^m, the sphere S^m participates in some form of topological construction that gives a correspondence with the sphere S^n. In the case of the equatorial map, the correspondence lies with the sphere S^n being given by a topological construction involving the equatorial sphere S^{n-1} and the great circles S^1. Although the full sphere S^n and its equatorial sphere S^{n-1} have different $(r_n, n+1)$ tuples, for the equatorial map to correspond to the same sphere, the two must at least have the same ratio in invariants:

$$\frac{r_n^2}{n+1} = \kappa$$

So in order for the sphere S^{n-1} of the equatorial map to correspond to the sphere S^n, the ratio of the radii of the two spheres must be given by:

$$\frac{r_{n-1}^2}{r_n^2} = \frac{n}{n+1}$$

For the Electroweak Vacuum map $S^4 \to S^3$ the radius of the equatorial sphere is that of the isospin sphere, $r_3 = r_I$, and the great circles of S^4 correspond to hypercharge, r_4=r_Y. So the ratio is given by 4/5, and this gives the value for the ratio between the group and physical Weinberg angles as being:

$$R_W = \frac{\tan\phi_W}{\tan\theta_W} = \frac{r_3}{r_4} = \frac{r_I}{r_Y} = \sqrt{\frac{4}{5}} \approx 0.8944272$$

This gives the predicted value for the ratio between the hypercharge and isospin components of the charge generator, and so the physical Weinberg angle θ_W is given by:

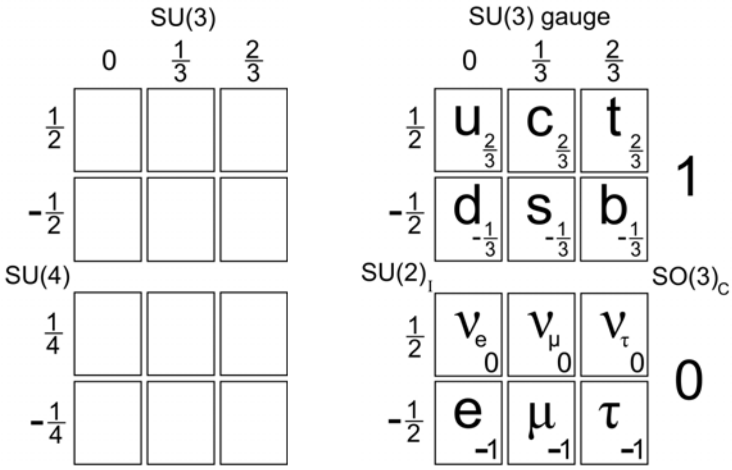
$$\theta_W = \tan^{-1}\left(\sqrt{\frac{5}{16}}\right) \approx \tan^{-1}(0.5590) = 29.2059°$$

This accounts for the apparent discrepancy between an irrational value for the Weinberg angle and the Electroweak Vacuum giving rise to topological monopoles. However, the transition will not just give rise to a single type of monopole, as there-exist a number of different ways of selecting the 4 dimensions of the electroweak base-space S^4 from the full S^7 particle space. For a trivial vacuum all the different ways will be equivalent, but for the non-trivial Electroweak Vacuum they will be distinct, and given by the homotopy group for the mapping of S^7 to S^4:

$$\pi_7(S^4) = Z\times Z_{12} = Z\times Z_3\times Z_4$$

The eigenvalues for the topological monopoles can be found from the unification "symmetry group" of SU(4)/SU(3) ≅ S^7, giving SU(3) group eigenvalues (0, 1/3, 2/3) and SU(4) group eigenvalues (–¼ , ¼, –½, ½), as shown in Figure 2a.

Figure 2: Monopole eigenvalues a) SU(4), SU(3) b) $SO(3)_C$, $SU(2)_I$, $U(1)_Q$



As SU(3) is not a normal sub-group of SU(4), the unification quotient SU(4)/SU(3) is not given by a symmetry group, but by the action of SU(4) over the differential manifold S^7. The equivalence between the particle dimensions of S^7 is broken by the Electroweak Vacuum, giving a symmetry breaking following the standard pattern for SU(n):

$$SU(4) \rightarrow SU(2) \otimes SU(2) \otimes U(1)$$

However, the group isomorphism SU(2) ≅ Spin(3) gives a choice between isomorphic symmetry groups:

$$SU(2) \otimes SU(2) \otimes U(1) \cong Spin(3) \otimes SU(2) \otimes U(1)$$

The group Spin(3) is the universal covering group for SO(3), and so the choice is between local colour symmetry groups $SU(2)_C$ and $SO(3)_C$. The selection is made on the basis of their colour eigenvalues, as $SU(2)_C$ has λ_c=±1/2, whereas $SO(3)_C$ has λ_c=0, ±1. Since the Electroweak Vacuum doesn't involve the colour-fibre, it is colourless λ_c=0, which picks out the local colour group as being $SO(3)_C$, and so the Classical Theory will have local symmetry group:

$$SO(3)_C \otimes SU(2)_I \otimes U(1)_Y$$

This gives the local colour symmetry group as being SO(3) – with covering group Spin(3) and a group space of S^3 – and not SU(3) as in the Standard Model. However, the Z_3 centre of the SU(3) symmetry in the "unification group" SU(4)/SU(3) affects the monopole eigenvalues, as the full unbroken symmetry of the coloured monopoles in this Classical Theory is not given by $SO(3)_C \otimes U(1)_Q$, but by:

$$\frac{\mathrm{Spin}(3)_C \otimes U(1)_Q}{Z_3}$$

The Z_3 factor gives coloured monopoles 1/3 electric charges, so that the charge eigenvalues are given by:

$$\lambda_Q = \lambda_I + \tfrac{1}{2}\lambda_Y \qquad \text{where} \qquad \lambda_Y = \begin{matrix} -1 \text{ for leptons} \\ \tfrac{1}{3} \text{ for quarks} \end{matrix}$$

which gives the eigenvalue spectrum for the topological monopoles shown in Figure 2b. The topological basis for this eigenvalue spectrum implies that it should apply to both electric monopoles and magnetic monopoles, as both can be topologically constructed around a spatial hole. When there-exists such a spatial hole inside which the Poincaré group of Relativity does not apply, the SU(2) rotation group must apply to the surface enclosing the hole. As discussed earlier, the SU(2) eigenvalues of ½ are given by the equatorial map of the S^3 rotation group space to the S^2 surface around the hole, and so the ½ spin of the object would also constitute a topological charge. So if the given spectrum topological monopoles are around such spatial holes, then they will have spin ½ and can be identified with the 12 fundamental fermions.

In an extension to General Relativity of the form of Kaluza-Klein Theory, the spatial holes of these topological monopoles could only be given by the particle dimensions being compactified from the radius of the particle space S^7 being on the same scale as the radius of space S^3 down to some compactifcation scale χ. Given the scale disparity between the cosmological scale of S^3 and the scale of the compactified particle space $\underline{S}^7$ (compactified dimensions will be underlined to distinguish them from non-compactified dimensions), some sort of compactification-inflation see-saw mechanism is required. The search for such a mechanism encounters the problem that radiation and the cosmological "constant" term have historically not been correctly accounted for in General Relativity.

Compactification-Inflation See-Saw

For a curved isotropic and homogenous space where the curvature can be completely characterised simply in terms of the difference between a volume in the space and the corresponding volume in flat space, i.e. by the Ricci scalar R, the metric field equations for the space are of the form:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{\kappa}{c^4} T_{\mu\nu}$$

The tensor $T_{\mu\nu}$ denotes the distribution of sources of curvature within the space, with κ being the coupling constant between the sources and the fabric of the space that is caused to curve, Λ is a cosmological constant, and c is the speed of metric waves within the fabric of the space. Imposing the conditions of homogeneity and isotropy onto the metric of a closed space gives a spherical space S^D with:

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-r^2} + r^2 d\Omega^2_{D-1}\right]$$

where Ω_{D-1} denotes a generalised angle element and $a(t)$ the radius of the sphere S^D. For a source tensor of the form of the stress-energy tensor in General Relativity with components $T_{tt} = \rho c^2$, $T_{ii} = p$, the field equations are:

$$\frac{D(D-1)}{2}\left[\left(\frac{\dot a}{a}\right)^2 + \frac{c^2}{a^2}\right] - \Lambda c^2 = \kappa\rho \qquad (G_{tt} \text{ component})$$

$$-(D-1)\left(\frac{\ddot a}{a}\right) - \frac{(D-1)(D-2)}{2}\left[\left(\frac{\dot a}{a}\right)^2 + \frac{c^2}{a^2}\right] + \Lambda c^2 = \frac{\kappa}{c^2} p \qquad (G_{ii} \text{ component})$$

General Relativity considers the case D=3, and the S^{10} Unified Field Theory (STUFT) will consider the case of D=10, but the equations also hold for the simpler case of D=2 where the acceleration equation (G_{ii}) becomes:

$$\frac{\ddot a}{a} = -\frac{\kappa}{c^2} p + \Lambda c^2$$

In this case it is clear that for any scenario where the term p decreases to zero with increasing radius a, the cosmological constant term Λ will dominate at large radius, and the equation then predicts that the surface tension will drive exponential inflation. The D=3 case of General Relativity also apparently possesses this characteristic, but the generality of a metric field theory allows this D=2 case to be related to a physical scenario where this conclusion is obviously nonsense. A 2D rubber surface enclosing a volume of gas can also be formulated in terms of a metric field theory describing the S^2 "cosmology". Whereas we can see such a physical balloon as the 2D surface of an object existing in 3 spatial dimensions – with radius a in the 3rd spatial dimension – any hypothetical inhabitants of this Balloon World would exist in only 2 spatial dimensions, and would describe the surface of their world by a local metric field theory of the form given.

An outside observer in 3D can see that the balloon is inflated to a 3D volume V by some gas at pressure P within the balloon, and that for adiabatic changes this gas would obey the 3D gas equation $PV = const.$ The gas pressure P pushing against the balloon surface will be resisted by the rubber material exerting a tension, i.e. a negative pressure $-p(a)$, within the surface. When the rubber balloon was inflated with gas, this negative pressure would have grown from zero to the value $-p(a)$, and the work done in stretching the rubber material would give it a positive energy density $\rho(a)$. This gives the diagonal elements of the stress-energy tensor for the rubber surface as being $(\rho(a), -p(a), -p(a))$, which has the same signature as that of the cosmological term $\Lambda g_{\mu\nu}$. Now to be a physically-real theory, all the terms of the metric field equations must denote something that physically exists, and this simple scenario of Balloon World indicates that the cosmological term Λ must be denoting the physical surface in order for it to be a physically-real term.

Without any sources of local curvature for the surface, the acceleration equation above only involves the cosmological term, and predicts that the increasing surface tension of the inflating rubber surface acts a source that drives the inflation of the balloon – which from our 3D perspective is obviously incorrect. To avoid such a scenario of unending inflation in Balloon World, the cosmological term would need to be a function of radius that decreases to zero as the radius increases. If the cosmological term gave the coupling of the rubber surface to the gas pressure inside, it would be of the form $\Lambda(a)=\Lambda_p p(a)$ where Λ_p is a pressure coupling constant. This does not violate the requirement that Λ be a "constant" because it varies with the extra-dimensional parameter a, but is constant with respect to variation within the surface – which is what matters as this is where the constancy condition comes from. If we were free to choose the value of Λ_p, setting $\Lambda_p = \kappa/c^4$ would then give a radial acceleration of zero. The radial velocity equation (G_{tt}) would then give:

$$\left(\frac{\dot{a}}{a}\right)^2 = \kappa\rho(a) + \Lambda(a)c^2 - \frac{c^2}{a^2} = \kappa\left(\rho(a) + \frac{p(a)}{c^2}\right) - \frac{c^2}{a^2}$$

which can possess zero velocity solutions for particular choices of $\rho(a)$ and $p(a)$. In this way, the hypothetical inhabitants of Balloon World could account for the surface of their "cosmology" expanding up to some size and stopping.

Allowing the cosmological term Λ to vary with the extra-dimensional variable a can be seen to yield a pressure term by considering the application of conservation laws to the field equations:

$$G_{\mu\nu} + \Lambda(a)g_{\mu\nu} = \frac{\kappa}{c^4}T_{\mu\nu}$$

The covariant derivative of the metric is zero in any basis, $g_{\mu\nu;\nu} = 0$, and that of the Einstein tensor is identically zero by the Bianchi identities, $G^{\mu\nu}_{;\nu} = 0$, and so for constant Λ the conservation law is $T^{\mu\nu}_{;\nu} = 0$. However, when the cosmological term is parameterised by the extra-dimensional variable a, the conservation law becomes:

$$\Lambda(a)_{;\nu}g^{\mu\nu} = \frac{d\Lambda}{da}a_{;\nu}g^{\mu\nu} = \frac{\kappa}{c^2}T^{\mu\nu}_{;\nu}$$

For an isotropic and homogenous physical space where $a(t)$ is a function of time, only the time derivative conservation equation (i.e. energy conservation) is modified:

$$\dot{a}\Lambda' = \frac{\kappa}{c^2}\left(\dot{\rho} - \frac{(\rho+p)}{V}\dot{V}\right) \qquad \text{which can be rewritten as} \qquad Vd\rho - (\rho+p)dV - \frac{c^2}{\kappa}Vd\Lambda = 0$$

to give the same form as the thermodynamic equation:

$$dE + PdV + VdP = 0$$

where the cosmological term corresponds to the VdP term.

In General Relativity, the stress term $T_{ii} = p$ in the field equations has been described as meaning that pressure acts as a gravitational source, which is highly misleading. For a gas inside some volume V, the pressure P of the gas is given by the force F exerted against the surface area A of the bounding surface, which for the gas in the centre of the volume is not a local description. As the force on the enclosing boundary is due to the momentum change of gas particles as they collide with the surface, the local description of pressure is as a momentum density. This is what the stress term $T_{ii} = p$ is denoting by way of pressure, and it is momentum density that acts as a gravitational source, not pressure as such. Whereas a positive pressure on the boundary surface A is locally given by a positive momentum density within the volume V, a tension on the boundary surface A must necessarily be given by a negative momentum density within the volume V, as for the cosmological term denoting the surface tension of Balloon World. Similarly, the work done against a tension will give the surface a local positive energy density, whereas the work done by a pressure will give a local negative energy density, as occurs for a negative cosmological term.

For a metric space with a boundary, the metric field equations will contain a boundary term which would necessarily have to account for any tension given to the surface of the metric space. But for a closed metric space, such as a closed S^D cosmology, there is no such boundary term, and the global pressure or tension being applied to the space is not being accounted for by the stress-energy tensor. The radiation pressure in the stress term $T_{ii} = p$ is a local term denoting momentum density as a gravitational source, and so it is not a pressure term as such. By setting the cosmological term to be $\Lambda(a) = \Lambda_p p(a)$, the radiation pressure is specifically being included as a pressure.

This outward pressure of metric wave radiation in a metric field theory can also be found by considering the scenario of wave modes within the surface of a 2-sphere in 3-dimensional space where all other dimensions are compactified to $\underline{S}^n$ at all points on the sphere S^2, so as to leave the interior of the sphere totally empty of space. Simple thermodynamic analysis shows that the contraction of such a sphere would exclude metric wave radiation from the surface of the sphere into the surrounding 3D space, where the radiation energy density has the same characteristic relation $\rho = 3\kappa T^4$ as for a black body at temperature T. It is similarly easy to show [1] that show that the metric wave radiation within such a 2-sphere will give rise to a pressure and radial pressure gradient given by:

$$p_2 \propto \frac{1}{r^3} \qquad\qquad \frac{\partial p_2}{\partial r} \propto \frac{1}{r^4}$$

Now the radiation emitted from the contracting 2-sphere will exert a radiation back-pressure on the 2-sphere, and if this back-pressure P is equated with the change in metric wave pressure with radius, $P = \partial p_2/\partial r$, we find that the apparent temperature T of the black body in 3-dimensions is inversely proportional to the radius, $T \propto 1/r$, as for a Black Hole. The continuation of this thermodynamic analysis given in [1] showed that a compactified sphere of radius A has entropy:

$$S_2 = \frac{k_B}{(2\chi)^2} A$$

(setting χ to the Planck length gives the entropy expression for a Black Hole)

where the entropy S_2 has been given a dimensional suffix because the definition of entropy changes with the number of dimensions when there is dimensional compactification. Equating entropies defined in different numbers of dimensions will give an entropy anomaly, which underlies the information paradox of a Black Hole and why a cyclical universe with dimensional compactification does not violate the 2nd Law of Thermodynamics [1].

For this simple compactified sphere model of a Black Hole, the 2D pressure gradient is unopposed and leaks out into 3D space as the 3D radiation pressure P_3, which in thermodynamics is described in terms of a heat flow dq:

$$dE + PdV = dq$$

In contrast, the 2D cosmology of Balloon World had no space outside of it into which radiation could leak, and so energy conservation requires the insertion of a cosmological pressure term to take account of this radiation pressure gradient in the radial direction. The same will be true for an S^D cosmology in any number dimensions, including the spheres of the topological transition $S^{10} \to S^3 \times S^7$.

The simplest approximate model for the torus $T^{3+7}=S^3{\times}S^7$ is to link a S^3 cosmology to a S^7 cosmology by suitably constructed cosmological terms to take account of the relative radiation pressures. Assuming that the energy density of radiation scales as $p_3(a) \propto a^{-4}$ and $p_7(b) \propto b^{-8}$ in S^3 and S^7 respectively, where the scale factor of S^3 is $a(t)$ and that of S^7 is $b(t)$, the radiation pressure gradients leaking out from one space of the torus into the other would be:

$$\frac{\partial p_3}{\partial a} \propto \frac{1}{a^5} \propto P_7 \qquad\qquad \frac{\partial p_7}{\partial b} \propto \frac{1}{b^9} \propto P_3$$

For the scenario where the topological transition $S^{10} \to S^3 \times S^7$ occurs during the contraction of S^{10}, the contraction would be expected to initially carry over into $S^3{\times}S^7$, where it would lead to the 7D radiation pressure gradient increasing faster than that of the 3D radiation pressure gradient, so that the 7-to-3 pressure would be greater than the 3-to-7 pressure, i.e. $P_3{>}P_7$. This relative pressure difference would imply a net transfer of metric wave radiation from the shrinking S^7 to S^3.

In the S^7 cosmology, the radiation loss decreases the radiative pressure resistance to further contraction, but the velocity $\dot{b} < 0$ and acceleration $\ddot{b} < 0$ both remain negative and the S^7 continues to deflate. Whereas in the S^3 cosmology, the extra pressure from the radiation transfer first reverses the sign of the acceleration $\ddot{a} > 0$, then the velocity $\dot{a} > 0$, and this reversal of S^3 contraction into S^3 inflation gives the required compactification-inflation see saw of $\dot{b} < 0$ and $\dot{a} > 0$. When the transfer of radiation from the contracting 7-sphere to the expanding 3-sphere is complete, such that $P_3 = P_7$, the energy density of matter will dominate the pressure effect, and the sign of the acceleration will turn negative $\ddot{a} < 0$, as for a S^3 cosmology with no compactified dimensions and cosmological term. So the spatial S^3 cosmology will expand up to some maximum radius and then start to contract. If the radiation pressure of S^7 is low at that point, and both the acceleration and velocity are very small, then the 3-to-7 radiation pressure of the contracting S^3 cosmology would reverse the sign of the acceleration $\ddot{b} > 0$ and cause the compactified S^7 to reflate. The see-saw would then operate in reverse $\dot{b} > 0$ and $\dot{a} < 0$ with the contracting S^3 actively expanding the S^7, until the scale factors were comparable again $a \approx b$. At that point the topological transition could be reversed $S^3 \times S^7 \to S^{10}$, and so complete a full cycle of the cosmology.

For the radiation gas equation in 10D of $P_{10}V_{11}=const$, an equal pressure condition becomes equivalent to an equal volume condition, and this can be used to calculate the extent of cosmological inflation due to the compactification-inflation see-saw alone. For the constant volume condition $a^3b^8 = const.$ being applied to the see-saw of the torus T^{3+7}, the scale factor b giving the radial scale of S^7 is the physical scale χ against which all physical measurements are made. So the initial value of b in units of χ is b=1, and for a torus formed by the insertion of a hole in sphere the initial scale factors are the same $a = b = 1$. If the final value of $b=\chi$ is given by the Planck length l_p, then in S.I. units the measurable amount of spatial inflation due to the see-saw mechanism alone will be given by $a/b = (l_p)^{-11/3} = 1.252\times10^{129}$. This predicts that the compactification of the particle dimensions to the Planck scale will be accompanied by spatial inflation of the S^3 cosmology by a measured linear factor of 10^{129}, where roughly 10^{94} of the increase is due to a real spatial increase, and the remaining 10^{35} is due to the units used to measure physical distances decreasing, i.e. b compactifies to χ.

No Singularity Condition

The compactification-inflation see-saw following the topological transition $S^{10} \to S^3 \times S^7$ gives a cosmological scenario with an initial S^{10} cosmology with a maximum radius (the natural t=0 for the cosmology), and an S^3 cosmology also with a maximum radius. This scenario is necessarily cyclical ($\cancel{S}^1$) between the maximum limits of S^{10} and S^3 with no cosmological singularity – the minimum radius in the cosmology would be given by χ. This minimum radius implies a no singularity condition, because if any singularity were to form during the compactified phase whatever was inside the sphere of radius χ could not be recovered on the repeat cycle of the cosmology. So in order for the cosmology to be cyclical without loosing energy on each cycle, genuine singularities would be required to never arise – not inside a topological monopole nor a large scale Black Hole.

In 3+1 dimensions, an S^2 surface enclosing a spatial hole must be describable in terms of a representation of the rotation group in the rest frame of the sphere, and so the surface configuration of a topological monopole about the hole could not be unwound in solely 3+1 dimensions by any of the symmetry operators of the Poincaré group, which would imply that the configuration must possess a singularity. However, for space with $\underline{S}^n$ compactified particle dimensions, this conclusion no longer stands at the compactification scale as there is no topological barrier to an $S^2\times\underline{S}^n$ configuration being unwound by the symmetry operators of the hyperspace-time, such that the singularity is avoided. So in 10+1 dimensional hyperspace-time, the compactified 2-sphere $S^2\times\underline{S}^n$ could be realised where there was no singularity, because the spatial S^2 configuration can be unwound at the compactification scale within the surface of the sphere.

Such a realisation of the no singularity condition in the full hyperspace-time requires the existence of a compactified surface, which for a Black Hole in the 3+1 dimensions of General Relativity imposes the condition that there-exists a real event horizon surface. This is only an issue when the Black Hole is rotating, so consider the Kerr metric for a rotating Black Hole with mass m and angular momentum j in natural units G=1, c=1:

$$ds^2 = -\frac{\Delta - a^2\sin^2\theta}{R^2}dt^2 - 2a\frac{2mr\sin^2\theta}{R^2}dt\,d\phi + \frac{\left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta}{R^2}\sin^2\theta\, d\phi^2 + \frac{R^2}{\Delta}dr^2 + R^2d\theta^2$$

where

$$a \equiv \frac{j}{m} \qquad \Delta \equiv r^2 - 2mr + a^2 \qquad R^2 \equiv r^2 + a^2\cos^2\theta$$

The horizon for the rotating Black Hole occurs at $g_{rr}=\infty$, i.e. $\Delta = 0$, which in physical units is given by:

$$\Delta = r^2 - \frac{2Gm}{c^2}r + \frac{j^2}{m^2c^2} = 0 \qquad r = \frac{Gm}{c^2} \pm \sqrt{\frac{G^2m^2}{c^4} - \frac{j^2}{m^2c^2}}$$

The radius of the event horizon will only have real-valued solutions for angular momentum given by:

$$j \le \frac{Gm^2}{c}$$

For angular momentum greater than this bound, there will be no Black Hole solution with a real valued radius for the event horizon, and so the compactification condition that avoids the singularity could not be realised. The event horizon radius corresponding to this angular momentum bound is given by:

$$\chi = \frac{Gm_\chi}{c^2} \qquad \text{for which the angular momentum bound can be expressed as} \qquad j_\chi = \frac{c^3}{G}\chi^2$$

For a cosmology where the compactification scale χ is the smallest physical scale, this would give the basic value of angular momentum.

To demonstrate the effect of this, consider a mass travelling along the surface of a compactified tube inside which the Poincaré group does not apply. By Poincaré invariance, the surface of the tube has a rotation group eigenvalue of 1 corresponding to the basic angular momentum of j_χ. The mass m travelling along the surface of the tube will have linear momentum $p_z = mv_z$ parallel to the compactified tube, and angular momentum $j = m\chi v_\phi$ around the tube. As the angular momentum j is being set at the value of j_χ, whereas the mass m is indeterminate, the mass will be replaced by the angular momentum, and the angular velocity $v_\phi = \chi\omega$ then used to eliminate χ:

$$p_z = \frac{j_\chi v_z}{\chi\, v_\phi} = \frac{v_z}{v_\phi^2} j_\chi \omega$$

Considering the limit of $v_z = v_\phi = c$ for a circular wave mode around the compactified tube with angular momentum j_χ gives:

$$E = p_z c = j_\chi \omega$$

From this we have the obvious identification $\hbar = j_\chi$, with χ being the Planck length l_p and m_χ being the Planck mass m_p:

$$\hbar = m_p l_p c = \frac{c^3\chi^2}{G} \qquad \chi = l_p = \frac{Gm_p}{c^2} = \sqrt{\frac{\hbar G}{c^3}} \qquad m_\chi = m_p = \sqrt{\frac{\hbar c}{G}}$$

This identification $\hbar = j_\chi$ would appear to give the rotating Black Hole a rotation group eigenvalue of 1. However this would not be the measured angular momentum because a Black Hole rotating about the z-axis causes an angular rotation of any reference frame in the vicinity of the Black Hole given by (in dimensionless units):

$$\omega(r,\theta) = \frac{d\phi}{dt} = \frac{g^{t\phi}}{g^{tt}} = \frac{2Jr}{\left(r^2 + a^2\right)^2 - a^2\Delta\sin^2\theta}$$

At the surface of the event horizon, this frame-dragging for a Black Hole at the angular momentum bound is given by:

$$\omega(\chi) = c/(2\chi) \qquad \text{or} \qquad v = \omega\chi = ½\, c$$

Whereas the angular momentum bound given above corresponds to $j_\chi = m_\chi\chi c$ in the Kerr metric, the effect of frame-dragging will give the measured angular momentum of the horizon as $j = ½\, m_\chi\chi c$. Now the moment of inertia for a mass shell with mass m and radius r is $I_m = ½mr$, giving the angular momentum for rotational velocity v of $j=½mrv$. So the measured angular momentum $j = ½\, m_\chi\chi c$ of the event horizon would appear to be due to a mass shell m of radius χ rotating at the maximum velocity of c, giving a measured rotation group eigenvalue of ½. Note that all of the mass of the Black Hole residing in the surface of the event horizon is required by the compactification condition that avoids a spatial singularity.

For the $U(1)_Q$ topological monopoles, the unification of particle and spatial dimensions into S^{10} implies that the $U(1)_Q$ generator I_Q would be related to the rotation group generators J_i. This in turn implies that the physical scale factor $\hbar$ of the rotation group will be transferred to the $U(1)_Q$ group and so give a physical scale to the Dirac quantisation condition:

$$eq_m = \tfrac{1}{2} n\hbar c$$

The transfer of the physical scale $\hbar c$ to the symmetry group $U(1)_Q$ for electric charges means that the unit circle S^1 of the group $U(1)_Q$ operating over the charged monopoles will be given by group elements of the form:

$$\lambda = \exp\left(i\frac{e}{\hbar c}\phi \right)$$

However, this does not quite give the scale factor between the compactified dimensions and the unit circle $\mathcal{S}^1$ of the $U(1)_Q$ symmetry group, as $\hbar$ gives the physical scale factor between the compactified particle dimensions and the unit sphere $\mathcal{S}^3$ of the SU(2) ≅ SO(3) rotation group. As the ratio of the areas of the unit spheres $\mathcal{S}^1$ of $U(1)_Q$ and $\mathcal{S}^3$ of SU(2) is given by:

$$\frac{A_1}{A_3} = \frac{2\pi(r=1)}{2\pi^2(r=1)^3} = \frac{1}{\pi}$$

the scale factor between the compactified dimensions and the unit circle $\mathcal{S}^1$ of $U(1)_Q$ requires this additional $1/\pi$ factor of the area ratio between the $\mathcal{S}^1$ of $U(1)_Q$ and the $\mathcal{S}^3$ of the rotation group.

Electroweak Scale

To avoid the spatial singularity of a 3+1 dimensional Black Hole, the manifold of a topological monopole must be given by $S^2 \times \underline{S}^7$ where the particle dimensions $\underline{S}^3 \times \underline{S}^4$ are unified as $\underline{S}^7$ within the S^2 surface. The radius of the horizon for the Planck mass m_p gives an expression for the compactification radius χ, which can be used to give an expression for the energy density ρ_7 of the compactified $\underline{S}^7$:

$$\chi = \frac{Gm_p}{c^2} \qquad\qquad m_p^2 = \frac{c^4\chi^2}{G^2} = \frac{c^4}{4\pi G^2}A_2 = \rho_7^2 A_2$$

Now the non-trivial Electroweak Vacuum is given by the whole manifold $S^3 \times \underline{S}^3 \times \underline{S}^4$ of the cosmology, where the symmetry of the S^7 particle space is broken such that the compactified particle space is given by the product space $\underline{S}^3 \times \underline{S}^4$, where both spheres can be expected to compactify to radius χ. So the total mass m_η of the Electroweak Vacuum can be expressed in terms of the mass density $\rho_{3,4}$ of the particle dimensions $\underline{S}^3 \times \underline{S}^4$:

$$m_\eta^2 = \rho_{3,4}^2 A_3$$

Since all 3 particle spheres have the same radius χ, the mass density $\rho_{3,4}$ can be related to the mass density ρ_7 through the areas of the spheres.

$$A_3 = 2\pi^2\chi^3 \qquad A_4 = \tfrac{8}{3}\pi^2\chi^4 \qquad A_7 = \tfrac{1}{3}\pi^4\chi^7 \qquad \rho_{3,4}^2 = \frac{A_3 \times A_4}{A_7}\rho_7^2 = 16\rho_7^2 = 16\frac{m_p^2}{A_2}$$

As the physical unit of distance measurement in the product space $S^3 \times S^7$ is χ, the measurable radius of the S^3 cosmology is $R\chi$. This gives the ratio of the areas of the S^3 cosmology and S^2 monopole as being:

$$\frac{A_3}{A_2} = \frac{2\pi^2(R\chi)^3}{2\pi\chi^2} = \tfrac{1}{2}\pi\chi(R^3)$$

So in a local theory, the electroweak energy scale η of this global electroweak configuration will be given by:

$$\eta^2 = \frac{m_\eta^2}{R^3} = \left(\frac{A_3 \times A_4}{A_7}\right)\rho_7^2\left(\frac{A_3}{R^3}\right) = 16\left(\frac{m_p^2}{A_2}\right)\left(\frac{A_3}{R^3}\right) = 8\pi\chi\, m_p^2$$

For $\chi = l_p = 1.616199\times10^{-35}$ m and $m_p = 1.220932\times10^{19}$ GeV/c^2 this gives the electroweak scale as being $\eta = 246.0701$ GeV/c^2, which agrees well with the value in the Standard Model.

The Electroweak scale will enter into the metric field theory via a scalar term in the dimensionally reduced Lagrangian that holds after the compactification-inflation see-saw. Whereas there-exists a choice of gauge in Kaluza-Klein Theory such that the scalar field term ϕ appears as:

$$L_\phi = \tfrac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right)$$

such a gauge will not exist for the Electroweak Vacuum because of its non-trivial mapping to the spatial cosmology. This means that the maximum simplification possible by a suitable choice of gauge will be of the form:

$$L_\phi = \tfrac{1}{2}\left(D_\mu\Phi\right)\left(D^\mu\Phi\right) \qquad \text{where} \qquad D^\mu\Phi = \left(\partial^\mu + i\frac{g}{2\hbar c}\sigma_i W_i^\mu + i\frac{g'}{2\hbar c}X^\mu\right)\Phi$$

From the definitions of the charge generator and the electromagnetic gauge field:

$$Q = \sin\theta_W I_3^W + \cos\theta_W Y \qquad\qquad A^\mu = \sin\theta_W W_3^\mu + \cos\theta_W X^\mu$$

the isospin g and hypercharge g' coupling constants are related to the electromagnetic coupling constant e by:

$$g\sin\theta_W = g'\cos\theta_W = e \qquad\qquad \text{where} \quad \frac{e}{\hbar c} = \frac{1}{\pi}$$

With the predicted value of the Weinberg angle given earlier, $\sin\theta_W$ and $\cos\theta_W$ are given by:

$$\sin^2\theta_W = \frac{\tan^2\theta_W}{1+\tan^2\theta_W} = \frac{5}{21} \qquad\qquad \cos^2\theta_W = 1-\sin^2\theta_W = \frac{16}{21}$$

and so the predicted values of the coupling constants g and g' are given by:

$$\frac{g}{\hbar c} = \frac{e}{\hbar c\sin\theta_W} = \frac{1}{\pi}\sqrt{\frac{21}{5}} \approx 0.6523 \qquad\qquad \frac{g'}{\hbar c} = \frac{e}{\hbar c\cos\theta_W} = \frac{1}{\pi}\sqrt{\frac{21}{16}} \approx 0.3647$$

which compare with the corresponding values in the Standard Model of 0.652 and 0.357 respectively.

The Electroweak Vacuum with the same non-zero value $\Phi_0=\eta$ throughout the S^3 cosmology means that the scalar field term in the Lagrangian will give mass terms for the isospin gauge fields of:

$$m_W^2 = \left(\frac{g}{2\hbar c}\right)^2\eta^2 = \frac{21}{20}\left(\frac{e}{\hbar c}\right)^2\eta^2 \qquad\qquad m_Z^2 = \left(\frac{g'}{2\hbar c}\right)^2\eta^2 = \frac{21}{20}\times\frac{21}{16}\left(\frac{e}{\hbar c}\right)^2\eta^2$$

Using the expression for the Electroweak Vacuum predicts the W and Z masses to be:

$$\eta = \sqrt{8\pi\, l_p}\, m_p = 246.0701\ \text{GeV}/\text{c}^2$$

$$m_W = \sqrt{\frac{21}{20}}\left(\frac{e}{\hbar c}\right)\eta = \frac{\eta}{\pi}\sqrt{\frac{21}{20}} = 80.2608\,\text{GeV/c}^2 \qquad\qquad m_Z^2 = \sqrt{\frac{441}{320}}\left(\frac{e}{\hbar c}\right)\eta = \frac{\eta}{\pi}\sqrt{\frac{441}{320}} = 91.9503\,\text{GeV/c}^2$$

These compare with the current values for the W and Z boson masses of:

$$m_W = 80.398\ \text{GeV/c}^2 \qquad\qquad m_Z = 91.188\ \text{GeV/c}^2$$

In addition to the derivative scalar terms in the dimensionally reduced Lagrangian, if the scalar field term ϕ varies with space-time, then the spatial metric term $g_{\mu\nu}$ will have non-zero derivatives, and so the Christoffel symbols will have non-zero values. Such non-trivial Christoffel symbols will give non-zero terms in the Riemann tensor, and this is the origin of the scalar field terms of the Lagrangian in Kaluza-Klein Theory. However, for the non-trivial scalar field configuration of the Electroweak Vacuum, there will also exist non-trivial cross-terms between the compactified Electroweak base-space and the spatial dimensions. Such a cross-term can be seen in the field configuration of the cross-section of the monopole shown in Figure 1b, which is of the form $\Psi = \exp(i\theta)$. This is expressed in a co-ordinate independent way for topological winding number n as $\Psi = \exp(n\Phi.\hat{x})$. The spatial variation of the configuration in going around the space gives non-trivial spatial derivative terms of the form $n\Phi\Psi$ in the Christoffel symbols, and of the form $(n\Phi)^2\Psi$ in the Riemann tensor. This will give the corresponding scalar field Lagrangian of the form:

$$L_\phi = \tfrac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) + \tfrac{1}{2}\left(n\Phi\right)^2\phi^2$$

which is that of a scalar field with mass $m=n\Phi$. For the Electroweak Vacuum with $\Phi=\eta$ and topological winding number $n=½$, this will give the mass for the Classical Higgs field – and the quantised Higgs pseudo-particle [1] – as being:

$$m_H = \tfrac{\eta}{2} = \sqrt{2\pi\, l_p}\ m_p = 123.035\,\text{GeV/c}^2$$

Vacuum Reservoir Effect

For a Black Hole with an event horizon of the compactification scale $r_h = \chi$ at the angular momentum bound for a real event horizon radius, the radius of the ergo-region defined by $g_{tt} = 0$ is:

$$r_+ = r_h(1 + \sin\theta)$$

Outside of the ergo-region $r > r_+(\theta)$ the temporal metric term is of the same sign as for flat space-time, $g_{tt} < 0$, but inside the ergo-region $r_h < r < r_+(\theta)$ it is of the opposite sign, $g_{tt} > 0$. This reversal of the metric sign impacts the energy contribution made to the first Casimir invariant by any fields within the ergo-region:

$$C_1^{(3)} = g_{\mu\nu}P^\mu P^\nu = -|\,g_{tt}(r > r_+)\,|\,E_a^2 + |\,g_{tt}(r < r_+)\,|\,E_b^2 + \underline{P}\cdot\underline{P} = -m^2c^4$$

As discussed earlier, the frame-dragging for such a Black Hole is very significant close to the event horizon, and a further consequence of this is that radiation can become trapped in orbit about the Black Hole within the ergo-region. The dimensional compactification of the hyperspace-time of STUF-Theory means that the energy of all radiation is given by $E=\hbar\omega_p$, which for ω_p being given by the inverse of the Planck time $t_p = l_p/c$ is:

$$E = \hbar\omega_p = \hbar\frac{c}{l_p} = (m_p l_p c)\frac{c}{l_p} = m_p c^2 \qquad \text{using the definition of } \hbar$$

Since the energy of the Black Hole mass is $E = m_pc^2$, this would imply that radiation trapped within the ergo-region could potentially cancel all the mass contribution of the Black Hole, leaving the mass of the monopole largely determined by the far-fields due to the charge residing on the horizon.

The reversal of the temporal metric term within the ergo-region means that the form of metric wave radiation changes as it leaves the ergo-region, such that the wave radiation trapped within the ergo-region possesses non-propagating wave modes outside of the ergo-region. In the far-field limit $r \to \infty$ of a flat metric $\eta_{\mu\nu}$ in 3+1 dimensions, the first Casimir invariant for these non-propagating wave modes will be given by:

$$C_1^{(3)} = -E^2 + \underline{p}^2 = 0$$

So for the full wave modes from $r = r_h$ to $r \to \infty$, the contribution from the part of the wave inside the ergo-region means that in the far-field metric in 3+1 dimensions, the first Casimir invariant for these waves has:

$$C_1^{(3)} = -E^2 + \underline{p}^2 = -m^2c^4 > 0$$

As $m^2 < 0$, the radiation trapped within the ergo-region has the appearance of being a virtual radiation field in the metric of flat space-time. So instead of the rest mass of the monopole decreasing through the emission of propagating radiation – as for a large scale Black Hole – it is decreased through the emission of virtual radiation, i.e. radiation trapped within the ergo-region.

The extremely small size of the compactification scale means that all scientific theories of the topological monopoles of STUFT will be constructed in the far-field metric of the dimensionally reduced space-time of Relativity, because this is the space in which all observers exist and make experimental measurements. So in order to connect with the experimental measurements possible in space-time, a far-field formulation of the topological monopoles will be required to be based upon a virtual radiation field around an object of the Planck mass. A classical approximation of the topological monopoles of STUFT is given by assuming that the virtual radiation field cancels all the mass of the Black Hole, leaving the mass of the monopole being given by its field self-energy outside of some radius r_0. For an electron, the classical electron radius r_0 gives this boundary where $m^2 \approx 0$ and the full approximation is:

- Black Hole of mass m_p, radius l_p and net angular momentum of $½\hbar = ½m_pl_pc$
- Virtual radiation from $r = l_p$ to the classical electron radius r_0 with $m^2 = -m_p^2c^4$
- Electric field for $r \geq r_0$ so electron mass m_e given by $m_ec^2 = e^2/(4\pi\, r_0)$

However, this classical approximation will only be good for a single Black Hole monopole, because when there are multiple monopoles the virtual radiation tails outside of the ergo-regions will overlap with each other. Since the monopole charge and spin are both topological charges in any scientific theory of the monopoles, and the Planck mass of the Black Hole is also fixed, the far-field formulation of multiple monopoles will necessarily involve some form of virtual field expansion about the interacting monopoles. Furthermore, the topological defects occur as both monopoles and anti-monopoles and the formation of a monopole/anti-monopole pair from metric wave radiation of sufficient energy will be possible within the full metric field theory of STUFT. Such processes would also need to be included in any physically-real scientific theory of interacting monopoles in the far-field limit of flat space-time. Since the virtual radiation field has the energy of the Planck mass, but the rest masses of the monopoles are much less than this due to the cancelling effect of virtual radiation, the virtual field expansion of the far-field formulation would have to also include virtual matter terms as well.

In physically-real terms in the far-field limit of flat space-time, both the virtual matter fields and the virtual radiation fields will need to be expressed as propagating terms, but there is no net radiation from the monopole configuration. The only way to resolve this difference in physically-real terms is to express the virtual field expansion as reactions in terms of propagating objects and waves, where they appear in a particle reaction as propagating at some time t_1 at spatial location $\underline{x}_1$ and then disappear in another particle reaction at some time t_2 at spatial location $\underline{x}_2$. However, just as the sign reversal of the metric in the ergo-region leads to the energy anomaly $m^2 < 0$ of virtual fields in the 3+1 dimensional flat metric, it also gives a distance anomaly for events in the virtual field expansion where $ds^2 > 0$. Specifically, the space-time separation of two events involving the propagation of virtual matter or radiation just mentioned will be space-like.

This means that any far-field formulation of interacting monopoles in terms of a virtual field expansion will necessarily include the following conditions defining a Vacuum Reservoir Effect:

1) Energy Anomaly
2) Number conservation of monopoles
3) Distance Anomaly

The significance of this is that [1] showed that adding such a Vacuum Reservoir Effect to any physically-real scientific theory of a collection of experimentally observed particle reaction vertices renders the theory mathematically incomplete – all the steps of Gödel’s Incompleteness Theorem can be constructed in strictly physically-real terms within the scope of the theory. So any physically-real scientific theory of the topological monopoles of STUFT constructed in the far-field of space-time will be mathematically incomplete. The incompleteness result means that there-exists true statements which can be expressed in strictly physically-real terms within the scope of the theory, but cannot be derived within the theory. As discussed in [1] and [2], the use of strictly physically-real terms to denote a consistent reality will result in scientific theories that are known to be consistent, and so if they constitute one of Gödel’s “related systems” then they are proven to be incomplete. Since all experimental observations must necessarily be expressed in the same physically-real terms, it is possible that an observational statement could be such a non-derivable statement. This is experimentally verified to be the case as particles possess a wave property – stated in the physically-real terms of any scientific theory describing the interaction of particles with wave radiation – which cannot be derived in any consistent theory of classical physics.

Since Gödel’s Incompleteness Theorem explicitly requires the formal system in question to include arithmetic over the natural numbers, changing the physically-real terms for the natural numbers of monopoles to real number valued terms avoids the incompleteness result. However, this change results in non-physically-real terms that must also include the undecidable wave property found by experimental observation. Imposing the symmetry invariance of STUFT onto a scientific theory will result in a scientific theory that denotes the natural number of monopoles as continuous real number valued fields that are expanded in terms of wave modes (the undecidable property). The dimensional compactification to space-time gives the required spatial invariance as being that of the Poincaré group, with the monopoles being in the spinor representation with spin $\frac{1}{2}\hbar$ – where Planck’s constant $\hbar$ is defined as the angular momentum bound for a real event horizon in the Kerr metric so as to satisfy the no singularity condition of STUFT. The physical scale of $\hbar$ given to physical rotations propagates throughout the Poincaré group to give the infinitesimal rotation and translation operators as being:

$$J_z = i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) \qquad P_z = i\hbar\frac{\partial}{\partial z} \qquad E = P_0 = i\frac{\hbar}{c}\frac{\partial}{\partial t}$$

which adds the physical scale $\hbar$ to the commutator relation of Hamiltonian physics:

$$[x_\mu, P_\nu] = [x_\mu, i\hbar\partial_\nu] = i\hbar\delta_{\mu\nu}$$

The Heisenberg uncertainty relations trivially follow from this introduction of the physical scale factor $\hbar$ and the wave property for the fields. They also follow from the minimum angular momentum bound being $\hbar$, and in more general terms from the compactified dimensions defining the physical scale of the units of measurement for mass, length and time as being m_p, l_p and t_p, where:

$$m_p = \sqrt{\frac{\hbar c}{G}} \qquad l_p = \sqrt{\frac{\hbar G}{c^3}} \qquad t_p = \sqrt{\frac{\hbar G}{c^5}}$$

The symmetry breaking of the Electroweak Vacuum gives the local gauge invariance of the monopole spectrum (Figure 2b) as being SO(3)⊗SU(2) ⊗U(1), with the isospin, hypercharge and electric coupling constants as given earlier. As the colour space and the rotation group space are both S^3, there is no factor of $1/\pi$ as for the S^1 of the electromagnetic group, and so the colour coupling constant of the strong nuclear force is given by $g_s/(\hbar c) = 1$.

The global origin of the Electroweak Vacuum means that it cannot be directly denoted in a local theory, and so some invariant way of giving the value $\Phi=\eta$ for the electroweak scalar term of the dimensionally reduced metric is required. The simplest way to realise a non-zero value in a locally invariant theory is the introduction of a Higgs potential term:

$$V(\Phi) = \tfrac{1}{2}\mu^2\Phi^2 - \tfrac{1}{4}\lambda\Phi^4 \qquad \text{with minimum at } \eta = \sqrt{\frac{\mu^2}{\lambda}}$$

where the Higgs pseudo-particle mass of $m_H = \sqrt{(2\mu^2)}$ in the theory requires λ=1/8 for compatibility with the full metric field theory. This origin for the Higgs potential makes it an artificial feature of the local theory that is nonetheless required in order to account for the Electroweak Vacuum that arises at the global level of a non-trivial mapping from the electroweak base-space all the way around the whole S^3 cosmology. The fact that it involves a spatial mapping, allows the two options of $\pm\pi$ twist to be given the parity labels of left and right [1], so that Z_2 = {L, R}. Experimental observation gives the Electroweak Vacuum as being L.

As all the terms in a metric field theory are squared, the metric field theory in the S^{10} phase will be strictly $P_{10}T$ invariant, where a dimensional suffix has been added because compactification changes the number of dimensions to which the parity operator applies from 10 to 3. In any scientific theory over the monopoles that result from the Electroweak Vacuum, the parity operators associated with the 7 compactified dimensions become charge reversal operators $C(q) \rightarrow -q$. The global $P_{10}T$ invariance of the S^{10} phase becomes global C_7P_3T in the compactified phase, where the global invariance also requires a change in Electroweak Vacuum L → R. So any local theory for a cosmology with a given Electroweak Vacuum will only be C_4P_3T invariant, where C_4 runs over the colour and electromagnetic charges.

A clear distinction exists between C_4P_3T invariance applying to locally occurring events in a static background metric, and the time dependent global metric of an expanding cosmology with a non-trivial Electroweak Vacuum. In the latter case, the time reversal operator T will reverse the sequence t_3–t_2–t_1 of locally occurring events to the sequence t_1–t_2–t_3, but it will not reverse the time development of the global background metric in which the observer is travelling in the positive time direction. The time development of the background metric gives a global time to the cosmology, and particle interactions with the non-trivial Electroweak Vacuum in the expanding cosmology will give T violation, which by global C_7P_3T invariance implies local C_3P_3 violation due to the Electroweak Vacuum, i.e. the colour and electromagnetic interactions will be CP invariant, but the Electroweak interactions will not.

As the energy operator E in the background metric of the cosmology is defined in terms of the temporal translation operator P^0, the positive time bias of an expanding cosmology in the positive time direction implies a positive energy bias. As E>0 defines matter and E<0 defines anti-matter, T violation in the positive time direction implies a bias in favour of matter over anti-matter, and hence a net matter content within the cosmology. This effect would be strongest just after the end of the compactification-inflation see-saw when the topological monopoles formed, and the expansion rate $\dot{a} > 0$ was at its highest value in the compactified phase. As the expansion rate slows, the positive time bias of the expanding cosmology would decrease. It should be noted that whereas the view of anti-matter as being matter travelling backwards in time is an optional interpretation in the Standard Model, in a pure metric field theory such as STUFT it has to be the reality.

Discussion

The results of STUFT are obtained on the basis of topology, the geometric relationships of spheres and the no singularity condition in 10+1 dimensions translating into a real event horizon condition in 3+1 dimensional space-time. This means that STUFT is a class of metric field theory without arbitrary fields where the conditions of homogeneity and isotropy give the set of possible metric field equations $\mathcal{F}=\{F_\alpha(G, c, \Lambda_p),\ldots\}$. The leading candidate is the form of the Einstein field equations, which are derived on the basis that the curvature term in the action is given by $f(R) = R$, whereas the most general form of curvature term is $f(R,R_{\mu\nu}R^{\mu\nu},R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau})$. The full range of possible functions gives the range of elements of the set $\mathcal{F}$ – the STUFT class of metric field theory in 10+1 dimensions. The set $\mathcal{F}$ of possible field equations is subject to a number of significant constraints, the first being that they must support the formation of a wormhole solution in an S^{10} cosmology as this topological transition breaks the symmetry of the space. Obviously they must support the compactification-inflation see-saw mechanism given earlier and reduce to General Relativity in 3+1 dimensions in the compactified phase. In addition, they must support rotating Black Holes where the no singularity condition translates into a real event horizon condition with angular momentum bound given by:

$$\hbar = \frac{c^3}{G}\chi^2$$

This bound forms the basis for the definition of Planck's constant $\hbar$ and defines the fundamental units of mass, length and time m_p, l_p and t_p. Since the given definition of $\hbar$ yields the correct values, including those of the electroweak scale, the field equations are restricted to reproduce the same figures within the bounds of experimental error.

The initial energy density ρ_0 of metric wave radiation at time t=0 in the S^{10} phase gives the one physical parameter of the metric field theory $F_\alpha(G, c, \Lambda_p)(\rho_0,\ldots)$. It will determine the maximum radius of the S^{10} cosmology, the maximum radius of the S^3 cosmology, and the rate of expansion when the topological monopoles formed. As the rate of change of the scale factor $\dot{a}(t)$ gives the rate of time reversal violation, it will determine the balance between positive energy matter and negative energy anti-matter, and so determine the net matter content of the universe in the S^3 phase. These experimental observations will constrain the possible value of ρ_0.

The STUFT class of metric field theory essentially contains one variable w running over the set of possible wormholes $\mathcal{W}_\alpha = \{w_i, \ldots\}$ in each metric theory F_α. Each choice of metric field equations F_α and wormhole w_i defines a cosmology $C_{\alpha i}$:

$$F_\alpha(G, c, \Lambda_P)(\rho_0, w_i) \Rightarrow C_{\alpha i}$$

So the wormhole set $\mathcal{W}_\alpha$ for metric field theory F_α induces the cosmology set $C_\alpha=\{C_{\alpha i}, \ldots\}$, and the field equation subset $\mathcal{S}\subset\mathcal{F}$ defines the cosmology set $C = \cup\, C_\alpha$ of STUFT. As the S^{10} cosmology is cyclical, every element of $\mathcal{W}_\alpha$ for a particular metric field theory F_α could potentially be realised. It should be noted that this restriction to only one available degree of freedom w (essentially the time at which the wormhole forms) means that STUF-Theory will have very little room for fine tuning to fit observational measurements.

The STUFT assertion that the theory will only be consistent and complete if all the S^n spaces in the corresponding normed division algebras are physically realised, picks out the transition $S^{10} \to S^3{\times}S^7$ that yields $\{S^0, S^1, S^3, S^7\}$. As the consideration of the STUFT class of metric theory has specifically avoided detailed analysis of the metric field equations, it has not been shown that this transition is picked out in preference to the other possibilities with a twisted vacuum: $S^{10}{\to}S^2{\times}S^8$ or $S^{10}{\to}S^4{\times}S^6$. It should be noted that the set of spaces $\{S^0, S^1, S^3, S^7\}$ is a finite set because there are only 4 normed division algebras over the real numbers. This means that there can be no extension to STUFT by the addition of further spaces, and that the STUFT assertion uniquely specifies the topological transition $S^{10} \to S^3 \times \underline{S}^7$. The full sequence of STUFT is given by:

$$S^{10} \to_{CI} S^3 \times \underline{S}^7 \to_{CF} S^3 \times (\underline{S}^3 \times \underline{S}^4) \to_{BS} S^3 \times \underline{S}^3 \times (\underline{S}^3 \times \underline{S}^1) \to_{EW} S^3 \times \underline{S}^3 \times \underline{S}^1$$

where the transitions have been labelled as:

- CI: compactification-Inflation – de-unification
- CF: colour-fibre separation – GUT part I
- BS: electroweak base-space splitting – GUT part II
- EW: symmetry below the local electroweak scale η

This sequence naturally explains the apparent pattern of the running coupling constants, where in STUFT the 3 particle coupling constants are defined from $\hbar$, which is defined in terms of the gravitational coupling constant G. This unification of the 4 forces of physics and the fact that the STUFT assertion reproduces all the key features of the Standard Model of particle physics – specifically including the derivation of Quantum Physics – is significant evidence for the validity of the STUFT class of metric field theories. However, STUFT contains 2 very notable and unequivocal conflicts with the current state of physics:

- The spatial universe must be the closed sphere S^3
- The colour space is S^3, with local colour symmetry group SO(3), not SU(3)

The corrections to the handling of radiation in General Relativity are required for the compactification-inflation see-saw, but it should be noted that they are made on the basis of strictly using physically-real terms in classical physics. Such a careful approach to the formulation of scientific theories in physically-real terms is critical to the proof that classical physics is incomplete ([1] and [2]), and the origin of quantum theory [1]. This yields the unexpected condition that any theory of physics unification must prove itself to be incomplete, a condition that STUFT satisfies.

The unification of physics in a pure metric field theory means that the fabric of the 10+1 dimensional hyperspace-time is immeasurable because there is nothing else to measure it relative to. It also means that a spherical hole inside which the space does not exist is the only type of object within the space, i.e. all objects are Black Holes. A particle is at the lower limit of Black Holes where the final evaporation of the mass into propagating radiation is frustrated by the presence of topological charges (including spin ½) on the compactified surface of the event horizon. Instead, the mass is cancelled by a virtual radiation field in the ergo-region, where the far-field formulation gives the particle mass as being determined by the self-energy of the charge fields. This predicts the mass hierarchy of the electric monopoles with electric charge q_e and colour charge q_c as being:

$Mass(\lvert q_c\rvert > 0) > Mass(\lvert q_c\rvert = 0)$	coloured quarks heavier than leptons
$Mass(\lvert q_e\rvert = 2/3) > Mass(\lvert q_e\rvert = 1/3)$	top heavy quark doublet
$Mass(\lvert q_e\rvert = 1) > Mass(\lvert q_e\rvert = 0)$	bottom heavy lepton doublet

As the neutrinos possess both isospin and hypercharge, they will also have a very small far-field because the mass of the Z field is slightly greater than that of the W field, and so the fields due to the charges will possess a small region where they have yet to cancel out. So STUFT predicts non-zero masses for the neutrinos, and consequently that neutrinos (and anti-neutrinos) give a hot dark matter content to the cosmology.

In addition to this mass pattern, the masses of the monopoles could also be expected to increase with the increasing colour gauge of the families:

$$Mass(electron\text{-}family) < Mass(muon\text{-}family) < Mass(tau\text{-}family)$$

The mass spectrum of the fundamental particles fits this pattern except for an up-down quark mass anomaly where:

$$Mass(up) < Mass(down)$$

This would need to be accounted for in the quantum field theory that must be constructed as the classical physics theory of the monopoles is incomplete.

The Dirac quantisation condition for magnetic monopoles gives the magnetic charge strength as being much larger than the electric charge strength, which implies that the masses of the magnetic monopoles will be much larger than there electromagnetic duals:

$$Mass(q_m > 0) >> Mass(q_e > 0)$$

All variations of Grand Unified Theory yield magnetic monopoles and STUFT is no exception, despite the unification "symmetry group" of STUFT effectively being SU(4)/SU(3). Where these magnetic monopoles are and if they really occur as a complete dual set of 12 remain open questions.

The presence of an undecidable feature in the self-reaction formulation of a monopole implies that the masses and the transition rates between monopoles will be incalculable in any 3+1 dimensional formulation of monopoles. Whether the masses of the isolated individual monopoles are calculable in the full metric field theory in 10+1 dimensions is an open question. The interaction between monopoles in the full 10+1 dimensional theory could encounter problems with the consistent definition of terms (such as energy and charges) because of the change in the number of dimensions. This could render the calculation of the elements of the transition matrices between the families of particles incalculable even within the full 10+1 dimensional theory. In which case, the modified version of the Standard Model predicted by STUFT would still contain some parameters that could not be predicted, not because there-exists some other theory that could predict them, but because they are inherently not predictable.

[1] Michael James Goodband (2011), *Agent Physics*, M.J.Goodband (2012) [ISBN 978-0-9571490-0-7]
[2] Michael James Goodband (2012), *Incompleteness of Classical Physics*,
url: www.mjgoodband.co.uk/articles/Incompleteness.pdf