Incompleteness of Classical Physics

Abstract: This paper reviews the application of Gödel's Incompleteness Theorem [1] to Classical Physics given in Agent Physics [2], showing that there-exist conditions under which physically-real scientific theories can be proven to be mathematically incomplete.

The underlying basis for the application of physically-real scientific theories is the strict use of a physically-real denotation in the construction of a formal system L in mathematics. This can be expressed in predicate notation as:

$$Predicate(object) \Rightarrow value$$

where *object* is a physically-real term denoting some object, *value* denotes a physically-real value that can be measured by experiment, and the physically-real *Predicate* denotes the physical measurement process. This physically-real notation gives the following logical basis for a scientific theory:

- Logical truth values are given by existence: *true* when object *A* exists, and *false* when it doesn't
- Logical-negation: if *A* exists (*true*) then *not-A* is saying that *A* doesn't exist (*false*)
- Logical-and is given by conditional existence: *A and B* exist
- Logical-or is given by alternative existence: *A or B* exist

Any scientific theory will explicitly be a theory of causation, where causal events give the basis for logical-implication in physically-real terms:

If *A* and *A implies B*, then *B*

This says that if object state *A* is the cause of object state *B*, then if *A* exists at time t, *B* will exist at some later time $t + \Delta t$, and this gives the physical basis of *implies* in the formal system L of science. The identification of logical-induction first requires the introduction of a set theory to the formal system L, where physically-real predicates are used to classify objects into different types. Any set of objects S={*object*, …} defined by a physically-real predicate will constitute a well-defined set in the formal system L. It must be noted that the terms *object* and *S* denote different types in the set theory: *S* denotes a set whereas *object* denotes an *urelement*. This is different from mathematics where the empty set {} is used as the basis of set definition, and so the set theory of mathematics does not require a system of types, unlike the formal system L of science which does. This also means that logical-induction over the cardinality of some set *S* can proceed from n=0 in mathematics, but must always proceed from n>0 in the formal system of science:

> If a physically-real predicate *P* is true for a set containing one *urelement*, and it is also true for the successor function over the set, then the predicate *P* is true for the set no matter how many *urelements* it contains.

When there-exists n physically-real predicates that can be used to classify the objects of some physical system into n well-defined sets, the cardinality of these sets will give the set $X = \{x_1,\ldots, x_n\}$ of natural-number valued variables. Given the atomic basis of matter, the general form of a physically-real scientific theory in Classical Physics can be denoted as being $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$, where the variable $\underline{\mathbf{x}} = (x_1,\ldots, x_n)$ denotes the numbers of different types of object present, and the collection of physical parameters applying to the object system are denoted by $\boldsymbol{\rho} = \{\rho_1,\ldots, \rho_m\}$. The value of any physically-real predicate that applies to the object system will be predicted by the scientific theory in terms of some number theoretic function f over the variable $\underline{\mathbf{x}}$ and the physical parameters $\boldsymbol{\rho}$.

This gives the logical basis for the formal system L underlying any physically-real scientific theory of Classical Physics, where logical truth values are given by existence, logical-inference is realised through causation, and logical-induction is realised over well-defined sets such that the formal system L supports full logical deduction. The strict use of physically-real terms ensures that if reality is consistent, the formal system L will also be consistent. As logical truth values are given by existence, and the experimental observation is that an object state either exists or it doesn't (note that this is still true in Quantum Physics at the experimental level), reality is observed to possess the required form of consistency, such that the formal system L of science in strictly physically-real terms is **known** to be logically consistent.

As the set theory (with a system of types) included in the formal system L must support all the basic set operations, it will necessarily include arithmetic over the natural-numbers given by the cardinality of the sets. This means that the formal system L used in the construction of a physically-real scientific theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$ involving number-theoretic functions over the natural-number valued variable $\underline{\mathbf{x}}$, will meet the required conditions for the application of Gödel's Incompleteness Theorem. However, this does not necessarily prove that the physically-real scientific theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$ is incomplete because the incompleteness proof involves mathematical steps that may not correspond to anything physically-real. Although the set theory must support arithmetic, the actual operations of addition and multiplication won't be in physically-real terms within the theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$ unless they are implemented as physical causal events.

The basic operations of the predecessor function $p(\text{x}) = x-1$ and the successor function $s(x) = x+1$ will be in physically-real terms for any physical system that implements an object conversion process $A \to B$, as each conversion decrements the number of type A objects and increments the number of type B objects. If some collection of objects of type A are all converted to objects of type B, then the zero function $z(x) = 0\ \forall x$ will also be in physically-real terms for the number of objects of type A. The physically-real predicates used to classify the objects into sets by type can be used to give the projection functions, $P_i(x_1, \ldots, x_m) = \text{x}_i\ \forall x_1, \ldots, x_m$ in physically-real terms, which together with the zero function $z(x)$, predecessor $p(\text{x})$ and successor $s(x)$ functions would give the initial functions required for the recursive definition of number-theoretic functions to be in physically-real terms. However, the operation of addition requires the successor function $s(x)$ to be repeated a specific number of times N (not just some arbitrary number of times), and multiplication requires an M-repeat of an N-repeat of the successor function $s(x)$. In Classical Physics, such N-repeat control of some physical process would be expected to require a source of energy to power it. Entropy considerations also arrive at a similar conclusion, as any physical process that implements addition or multiplication over the object numbers will systematically increase the number of objects of some particular type, which would be expected to be associated with a decrease in entropy. As a systematic decrease in entropy would violate the 2nd Law of Thermodynamics, the conditions required for its derivation must be violated for a physical system to implement arithmetic over object numbers; so the quantity of energy in the physical system must not be conserved, and object state transitions within the physical system cannot all be equally likely.

This critical role played by an energetic N-repeat control process in the enablement of object arithmetic provides a basis for the following division of Classical Physics:

b: $a(s, e, rc) \to a', r$ $\qquad$ $e = 0$ Object Physics
$\qquad\qquad\qquad\qquad\qquad\qquad$ $e \neq 0$ Agent Physics

Where in this Agent Calculus notation the term a denotes an agent, s denotes a stimulus for the behaviour b, r denotes the agent behavioural response, rc denotes some resource that either promotes the behaviour or is consumed in the production of the response object r, and e denotes the energy supplied by an energy source. The lack of an external energy source in this definition of Object Physics means that there can be no energetic N-repeat control of another physical process, and so the conditions required for object arithmetic cannot be physically realised. This means that Gödel's Incompleteness proof cannot be expressed in strictly physically-real terms in Object Physics, and so no physically-real scientific theory of Object Physics can be proven to be incomplete.

Agent Physics allows for an external energy source capable of supplying the energy required for N-repeat control behaviour, and an external energy sink such as a heat bath capable of taking entropy out of the system. This combination would allow for the decrease in entropy associated with object arithmetic, and so there could exist physical conditions under which Gödel's Incompleteness Theorem can be applied to a physically-real scientific theory in Agent Physics. However, the conditions needed for object arithmetic are insufficient in themselves for the Incompleteness proof to apply.

Although every physically-real scientific theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$ will contain recursive number-theoretic functions describing the relevant physically-real predicates and physical laws of causation, the Incompleteness proof requires every possible recursive number-theoretic function to be expressible within the theory. This first requires the initial functions $z(x)$, $p(x)$, $s(x)$ and $P_i(x_1, \ldots, x_m)$ to be in physically-real terms, as well as some N-repeat control behaviour over $s(x)$ and object addition so that full object arithmetic is in physically-real terms. If the rules for creating new functions from existing functions are also in physically-real terms, then recursive number-theoretic functions not in the scientific theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$ can nonetheless be expressed in physically-real terms within the scope of the theory.

For the function creation rules to be in physically-real terms, there must be a collection of agents displaying different behavioural responses to different stimuli, such that the agents form a behavioural agent network (BAN). A function f is obtained from functions g, $h_1, \ldots, h_m$ by substitution when:

$$f(x_1, \ldots, x_m) = g(h_1(x_1, \ldots, x_m), \ldots, h_m(x_1, \ldots, x_m))$$

For this to be in physically-real terms, the responses of some agents must serve as the stimuli for other agent behaviours, such that the number of stimuli input into some agent modelled by $h_i(x_1, \ldots, x_m)$ in the scientific theory, then determines the number of stimuli x_i input into some other agent modelled by g in the scientific theory. The newly created function f is then modelling the agent-component formed by the agents (h_i) feeding their responses into the other agent (g).

This inter-agent linkage of the BAN further requires the creation of new agent and object types to be by agent action, so that function creation by recursion will be in physically-real terms within the theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$ describing the agent system. A function f is obtained from functions g and h by recursion when:

$$f(x_1, \ldots, x_m, 0) = g(x_1, \ldots, x_m)$$
$$f(x_1, \ldots, x_m, y+1) = h(x_1, \ldots, x_m, y, f(x_1, \ldots, x_m, y))$$

The newly introduced natural-number valued variable y must be the number of objects of a new type that can be classified into a set by some physically-real predicate, i.e. y is the cardinality of some new well-defined set. Furthermore, the new type of objects must act as stimuli for some pre-existing agent or agent-component described by some number-theoretic functions within the theory. Under such conditions, some new number-theoretic functions not already present within the theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\boldsymbol{x}})$ will be expressible within the scope of the theory. However, the application of the Incompleteness proof requires that **every** recursive number-theoretic function be expressed within the theory, which requires that the agent behaviours underlying the function creation rules must be capable of generating an unlimited range of new types of object. This can generically be satisfied by agent behaviour that creates a combinatorial entity, where the range of combinations is unlimited, the occurrence of each particular combination can be counted by the natural-numbers, and those numbers are unrestricted in magnitude.

At the most basic level of biology, this situation is realised by random variations over the combinatorial chemistry of RNA/DNA and polypeptides. Some of the polypeptides fold to becomes proteins that have chemical or physical actions powered by some type of fuel object (e.g. ATP or NADH), and so they constitute types of protein-agent. The actions of these protein-agents include object conversion reactions $A \rightarrow B$, where the objects serve as both stimuli and responses of various different protein-agents, and so the protein-agents form a BAN that will include a physical realisation of function creation by substitution within a physically-real scientific theory. The protein-agent actions include the construction and destruction of compound objects made of N subunits of different types, including the RNA/DNA templates and the polypeptide chains to which they are translated. As the DNA copying process is subject to random variation, the proteins translated from the genetic templates are also subject to random variation, as is the range of chemicals produced by protein-agent action. This random variation gives a physical realisation of the unlimited creation of new object types of unlimited number, which is required for unlimited recursive function definition to be in physically-real terms within the physically-real scientific theory of the gene-protein system. The critical N-repeat control required to enable object arithmetic can be physically realised either by the construction of some object made of N subunits that changes some property at N units, or by a phase transition characteristic at N molecules of some chemical subject to antagonistic protein-agent actions. Under these conditions, the physically-real scientific theory of the gene-protein system will constitute a formal system L that supports full logical deduction, arithmetic over the natural-numbers, and is such that every recursive number-theoretic function can be expressed in physically-real terms within the scope of the theory. This means that every step of Gödel's Incompleteness proof can be reproduced in strictly physically-real terms within the scope of the theory, including:

- the specification of the Gödel number g for any statement P expressible within the theory
- the diagonal function D
- the Gödel sentence G
- the Rosser sentence R

and so the theory can be proven to be mathematically incomplete. It should be noted that the Incompleteness proof requires the theory to be **known** to be consistent, because a corollary of Gödel's Incompleteness Theorem is that if the theorem applies, the theory cannot be mathematically proven to be consistent. The strict use of physically-real terms ensures that the experimental observation of the consistency of reality translates into a scientific theory that is known to be consistent, and so the application of the Incompleteness Theorem proves the theory to be mathematically incomplete.

A further corollary of Gödel's Incompleteness Theorem is that if a theory is incomplete, then no extension to the theory based upon more natural-number variables will change the incompleteness result. In terms of biology, this means that the proof of incompleteness for the physically-real scientific theory of the gene-protein system remains valid when the theory is extended with the introduction of natural-number variables for the number of cells in an organism, and the number of organisms in an ecosystem. This means that **all** physically-real scientific theories in biology are proven to be mathematically incomplete. The corollary also means that there is no hidden variable theory that is mathematically complete, as any imagined hidden variable would have to take natural-number values if it is to be physically-real. It should be noted that this elimination of hidden variable theories will hold for any physically-real scientific theory in Classical Physics to which the Incompleteness Theorem applies.

However, demonstrating that there-exist mathematical conditions under which Gödel's Incompleteness proof can be constructed within the scope of a physically-real scientific theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\boldsymbol{x}})$ doesn't prove that the mathematical incompleteness is of any scientific relevance. The formal system L underlying the scientific theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\boldsymbol{x}})$ is based upon a set of axioms P_A that includes the basis of logic, set theory and arithmetic over the natural-numbers. As a formal language, there-exists a set P of propositions that can be stated within L, and as a deductive system there-exists a set P_D of propositions that can be derived from the set of axioms P_A within the formal system. The application of the Incompleteness Theorem given above considers the case when all the elements of P_A can be expressed in physically-real terms, and so P_D constitutes the set of all the scientific predictions that can be made by the theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\boldsymbol{x}})$. Now Gödel's Incompleteness proof establishes that the set of all propositions P is not the same as the set of derivable propositions P_D by finding a proposition u (the Gödel sentence) that is an element of P but not of P_D. The fact that the Rosser sentence is a second such proposition u, establishes that there-exists a non-singleton set P_U of undecidable propositions in P that are not elements of P_D, such that $P{=}P_D \cup P_U$. If the formal system L is expressed in strictly physically-real terms, then the set of observations of the physical system can be expressed as a set of propositions P_O in the formal language L, where necessarily $P_O \subset P$. The theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\boldsymbol{x}})$ would be scientifically incomplete if there-exists an observation $p{\in}P_O$ that is not predictable within the theory $p{\notin}P_D$, which could occur when the theory is mathematically incomplete $P_U{\neq}\{\}$ as it is possible that $P_O \cap P_U{\neq}\{\}$.

As the common feature of the Gödel and Rosser sentences is self-reference, the only conjecture that they appear to provide any information for is that the elements of the set P_U may all contain self-reference - note that induction from two elements of a set of unknown size to a universal statement about the set would be logically invalid, hence it is only a conjecture. The character of logical deduction would seem to suggest the same conjecture. Logical deduction in a formal system L from a set of axioms P_A proceeds in a stepwise fashion, which is capable of traversing any linear sequence or branching network of propositions in the full space of propositions P. Such a stepwise procedure can also enter into logical cycles that possess a natural entry point which connects with the branching network emanating from the set of axioms P_A in the space of propositions P. However, there can exist formal systems that possess closed cycles which cannot be broken open to give a unique starting point without destroying their character.

Self-referential statements in general possess the same problem of not possessing a unique entry point for the stepwise process of logical deduction. In a physically-real scientific theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$ modelling causation within an agent system, any self-referential statement expressed in physically-real terms would necessarily be a statement about causal closure within the agent system. Such causal closure entails a self-consistent state where the dynamic state ψ of the agent system at time t acts as the cause for the agent system to be in the same dynamics state ψ at some later time $t+\Delta t$. This can be viewed in physical terms as a sustainability condition on the dynamic state of an agent system of freely acting independent agents, such that the physical conditions required for sustainability of the self-consistent state can be found from basic energy and object conservation laws. Adding in the physical conditions required for object arithmetic, and the generation of recursive functions in physically-real terms, gives the following conditions for physically-real scientific theories of an agent system to be mathematically incomplete:

1) Agents
2) Non-thermal energy source
3) Energy sink
4) Self-consistency in the energy-cycle
5) Object source
6) Object sink
7) Object type conversion behaviour
8) Self-consistency in the object-cycles, including between object source and sink
9) If the system possesses a physical boundary, that boundary is differentially impenetrable to objects depending upon object type
10) Successor function behaviour b_{+1}
11) Predecessor function behaviour b_{-1}
12) N-repeat behaviour b_n using some compound entity of the form C_n that acts as a set of n objects
13) N-repeat behaviour b_n over the successor behaviour b_{+1} to give additive behaviour $b_{+n}= b_n b_{+1}$
14) N-repeat behaviour b_m over additive behaviour b_{+n} to give multiplicative behaviour $b_{m\times n}=b_m\, b_{+n}$
15) Self-consistency in the construction-destruction cycle of the C_n
16) Agent creation
17) Agent destruction
18) Self-consistency in the agent creation-destruction cycle
19) Extendibility condition for recursive number-theoretic function definitions

Any proposition about the self-consistent state of the agent system $p\in P_{SC}$ would necessarily be about causal closure over the agent system, and would inevitably possess an element of self-reference. Given the earlier conjecture about undecidable propositions and self-reference, the scientific relevance of the mathematical incompleteness of a physically-real scientific theory $\boldsymbol{S}(\boldsymbol{\rho}, \underline{\mathbf{x}})$ of an agent system would appear to be predicated upon $P_{SC} \cap P_U \neq \{\}$.

If there-exists some collection of agents $A=\{a_1, \ldots\}$ that form a sustainable agent system which satisfies the above conditions, and it possesses some observed feature $p\in P_O$ of a dynamic self-consistent state $p\in P_{SC}$ that is undecidable $p\in P_U$ in the physically-real scientific theory of the agent system, then the theory would be scientifically incomplete. However, this would not mean that a complete scientific theory didn't exist, just that it couldn't solely be based upon physically-real terms. This can be seen by considering how to denote the collection of agents A when the dynamic self-consistent state Ψ of the agent system results in it acting as if it were a single agent a. The issue is that A denotes a set and the term a is an *urelement*, and so equating the two is a set-theoretic type conflict that is explicitly forbidden in a set theory with a system of types. If the agent system does not possess any undecidable features, this type conflict of denoting a set of agents A as the agent entity a will not actually occur in any experimental predictions made by a complete physically-real scientific theory of the agent system. This is because $P_U=\{\}$ and so all the propositions P that can be stated in the scientific theory can be derived from the set of axioms P_A, i.e. $P = P_D$. As the behaviours of the individual agents of A form the basis of the axiom set of the theory, every prediction of an observable feature will be expressible within the scientific theory in terms of the individual agents of A. So for this case, the term a will never actually appear in any experimental prediction for the agent system. However, this will not be case if there-exists a single observable feature that is undecidable $p\in P_U$ in the physically-real scientific theory as it cannot be predicted from the behaviours of the individual agents of A. The observed undecidable feature p would have to be attached to an *urelement* term a denoting the set of agents A as a single agent entity, in order to construct a complete scientific theory of the agent system. The term a is obviously not a physically-real term, and so a complete scientific theory of an agent system with an observable undividable feature must necessarily contain non-physically-real terms.

The stepwise character of the formal system L was emphasised earlier because Gödel's Incompleteness Theorem only holds for arithmetic over the natural-numbers, where the numbers are obtained by the stepwise application of the successor function $s(x)$. If the natural-number valued variables $\underline{\mathbf{x}}_{(n)}$ of a mathematically incomplete theory $\boldsymbol{S_n}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$ are replaced with real-number valued variables $\underline{\mathbf{x}}_{(r)}$, the corresponding theory $\boldsymbol{S_r}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(r)})$ is **not** proven to be mathematically incomplete. However, denoting a physical natural-number valued quantity as a real-number valued quantity results in a term that is not physically-real, and so the conclusion is the same: if Gödel's Incompleteness Theorem applies to the physically-real scientific theory of an agent system, then the complete scientific theory must necessarily be based upon non-physically-real terms.

As the non-physically-real variables $\underline{\mathbf{x}}_{(r)}$ are real-number valued, but the numbers of physical objects are natural-number valued and will still be denoted by $\underline{\mathbf{x}}_{(n)}$ in experimental measurements, some mathematical procedure M must be added to the complete scientific theory $\boldsymbol{S_r}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(r)})$ in order to make experimental predictions of the measurable quantities, $\underline{\mathbf{x}}_{(n)} = M(\underline{\mathbf{x}}_{(r)})$. The corollaries of Gödel's Incompleteness Theorem can be used to show that M cannot simply be a number-theoretic function, by noticing that if it was its inverse M^{-1} could be used to substitute for $\underline{\mathbf{x}}_{(r)}$ in the complete scientific theory $\boldsymbol{S_r}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(r)}= M^{-1}(\underline{\mathbf{x}}_{(n)}))$, to give a theory of the form $\boldsymbol{S'}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$. Now the theory $\boldsymbol{S_r}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(r)})$ is assumed to be the scientifically complete version of the incomplete theory $\boldsymbol{S_n}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$, where Gödel's Incompleteness Theorem proves that all theories over the natural-number valued variables are incomplete. As the theory $\boldsymbol{S'}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$ is of this form, it is proven to be mathematically incomplete, in contradiction to the conclusion of the substitution $\underline{\mathbf{x}}_{(r)}=M^{-1}(\underline{\mathbf{x}}_{(n)})$, and so the inverse M^{-1} cannot exist. Furthermore, if the required mathematical procedure M could be deduced within the physically-real scientific theory $\boldsymbol{S_n}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$ – which would be the case for a number-theoretic function – then its inverse M^{-1} could be deduced within the theory as well. Since the inverse M^{-1} cannot exist, the mathematical procedure M must not be deducible in physically-real terms within the scope of the theory $\boldsymbol{S_n}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$.

The same approach can be applied to any interpretation I of the non-physically-real variables $\underline{\mathbf{x}}_{(r)}$ that converts the real-number values directly into the natural-number values of $\underline{\mathbf{x}}_{(n)}$. The consistent physically-real theory $\boldsymbol{S_n}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$ is proven to be incomplete, and because Gödel's Incompleteness Theorem proves that no theory of the form $\boldsymbol{S'}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$ can be both consistent and complete, any interpretation I of the complete theory $\boldsymbol{S_r}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(r)})$ that yields natural-number valued quantities cannot also be consistent.

The foregoing analysis has deduced the following properties of any complete scientific theory $\boldsymbol{S_r}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(r)})$ created for an incomplete physically-real scientific theory $\boldsymbol{S_n}(\boldsymbol{\rho}, \underline{\mathbf{x}}_{(n)})$:

1) The theory contains one or more non-physically-real variables
2) The undecidable property, or properties, found by experiment have to be directly included in the non-physically-real variables as they cannot be deduced from the physically-real theory
3) The non-physically-real variables take continuous real-number values for the natural-number values of the numbers of objects
4) There is no hidden variable theory without the non-physically-real variables that can produce the same experimental predictions
5) One or more of the non-physically-real variables embody a set-theoretic type conflict, denoting a set as an *urelement*
6) A mathematical procedure M must also be introduced to convert the continuous real-number values of the non-physically-real variables into the physically-real natural-number values for the numbers of objects
7) This mathematical procedure M, or more specifically its inverse M^{-1}, cannot be deduced from any physically-real scientific theory of the objects in question
8) The theory contains genuine inconsistencies in the direct interpretation of the theory, but these do not appear in any experimental predictions of the theory

As these statements are known to be true of Quantum Theory, the natural place to look for the experimental verification of the scientific incompleteness $P_{SC} \cap P_U \neq \{\}$ of Classical Physics is in the construction of a physically-real scientific theory of particle reactions. The experimentally observed particle tracks of cloud chambers and bubble chambers, together with the experimental measurements of the basic properties of particles, provide the physically-real terms for the construction of a theory in Classical Physics. It should be noted that all experimental data is explicitly stated in physically-real terms, without reference to the non-physically-real terms of Quantum Theory. Therefore the construction of a physically-real theory of particle reactions can proceed on the basis of experimental evidence independently of Quantum Theory. The physical conditions given above for mathematical incompleteness are satisfied for such a Classical Physics theory if the following *Vacuum Reservoir Hypothesis* is included in the theory:

- A physical system with energy E and particle number N, can increase its energy by ΔE and increase the particle number ΔN during some physical process of duration $\Delta t>0$, such that both the energy and particle number return to the values E and N when the process has finished.

Adding the vacuum as a source of energy for particle reactions turns them from being objects in Object Physics into agents in Agent Physics, and their reactions then support object arithmetic where the vacuum state with particle number deficits serves as the combinatorial entity required to enable N-repeat behaviour. Particle reactions can be arbitrarily combined to form a particle reaction component that can then be included as a unit an arbitrary number of times in another particle reaction network. In this way, the function creation rules of substitution and recursion are realised in physically-real terms within the theory, where the unrestricted nature of the *Vacuum Reservoir Hypothesis* means that the definition of recursive number-theoretic functions is similarly unrestricted. Consequently all the steps of Gödel's Incompleteness proof can be expressed in physically-real terms within the scope of the theory, and so the physically-real theory of particle reactions in

Classical Physics is proven to be mathematically incomplete, i.e. $P_U \neq \{\}$.

The addition of the *Vacuum Reservoir Hypothesis* to Classical Physics given above, then says that every observable particle partakes in an infinite set of self-reactions that lead from the physical particle back to itself, and so every particle is a self-consistent state in the particle reaction theory, i.e. $P_{SC} \neq \{\}$. So if a particle possesses a single feature p that is undecidable in Classical Physics then it is experimentally verified that $P_{SC} \cap P_U \neq \{\}$, which is the case because particles possess a wave property. In Classical Physics, the classifications of particle and wave are mutually exclusive, such that a wave can alternatively be expressed as not-particle, and so wave-particle duality is a statement of the form *P* and not-*P*. For a consistent reality and a Classical Physics theory in strictly physically-real terms, such an inconsistency cannot be arrived at within the scope of the consistent theory and so a wave property for a particle is an undecidable feature. So Classical Physics is experimentally verified as being scientifically incomplete. Furthermore, replacing the physically-real variable $\underline{\boldsymbol{x}}_{(n)}$ with the non-physically variable $\underline{\boldsymbol{x}}_{(r)}=\Psi$ of a real-number valued field describing the infinite set of particle self-reactions gives a scientifically complete theory that displays the 8 characteristics given above. As these itemise the dominant unusual characteristics of Quantum Theory (further characteristics are deduced in [2]), and yet can be deduced within the context of Classical Physics in physically-real terms, it can be concluded that Quantum Theory is not fundamental.

Since there are no particle reactions involving only mass (i.e. not involving electric, isospin or colour charges) to which the *Vacuum Reservoir Hypothesis* can be applied, the theory of General Relativity is not proven to be mathematically incomplete within Classical Physics. As the replacement of $\underline{\boldsymbol{x}}_{(n)}$ by $\underline{\boldsymbol{x}}_{(r)}=\Psi$ forms the basis for constructing the complete scientific theories of particle-reactions (i.e. the Quantum Field Theories), the fact that General Relativity is not incomplete implies that it does not need to be quantised. Together with the conclusion that Quantum Theory is not fundamental, this implies that physics unification should be sought by extending General Relativity to include the particle symmetries. Such physics unification was demonstrated for a class of metric field theory, S^{10} Unified Field Theory (STUFT), in [2] and [3]. In STUFT, there is geometric quantisation due to the compactification of dimensions to the scale χ, with Planck's constant being defined within the Classical Physics of STUFT as $\hbar = c^3\chi^2/G$. In STUFT, particles have a topological origin as rotating black holes of the Planck mass m_p, with an ergo-region where radiation trapped within it has $m^2<0$ corresponding to virtual-radiation, and this effect cancels the mass of the black hole to 1st approximation. In STUFT, this virtual-radiation is the origin of a vacuum reservoir effect that reproduces the conditions of the *Vacuum Reservoir Hypothesis* given earlier. Consequently, any physically-real scientific theory of the topological monopoles can be proven to be mathematically incomplete within Classical Physics, where the complete version is necessarily of the form of a Quantum Field Theory. The unification of physics provided by STUF-Theory ([2] and [3]) verifies the earlier conclusion that Classical Physics in strictly physically-real terms is consistent, but mathematically and scientifically incomplete

The verification of the scientific incompleteness of Classical Physics ($P_{SC} \cap P_U \neq \{\}$) implies that the mathematical incompleteness of any physically-real scientific theory of biology proven earlier, is realised as scientific incompleteness for the dynamic self-consistent states of biological-agent systems, i.e. there-exists observable features of biological systems that cannot be derived in any physically-real scientific theory of biology. Outside of particle physics, a network approach to the behavioural agent network (BAN) of an agent system is more appropriate [2], with an activity based perspective of the node content and edge transmission of the BAN proving to be the most general formulation. In this network formulation it is found that Gödel's Incompleteness proof can be reproduced within the physically-real scientific theories of biology, psychology, socio-economics and finance at higher levels of modelling, i.e. above the level of the gene-protein system underlying all biological systems. However, such application of Gödel's Incompleteness proof within the scope of such theories has to be done carefully, as the simplistic approach is generically incorrect.

The derivation of Quantum Physics within Classical Physics shows that all of science is within the scope of Classical Physics, and so it should be apparent that the scientific and philosophical implications of these incompleteness results will be non-trivial and widespread. Agent Physics [2] outlines how the incompleteness result for Classical Physics forms the basis for a pan-science paradigm shift.

[1] Kurt Gödel (1931), *On Formally Undecidable Propositions of Principia Mathematica and Related Systems*, Dover Publications (1992)

[2] Michael James Goodband (2011), *Agent Physics*, M.J.Goodband (2012) [ISBN 978-0-9571490-0-7]

[3] Michael James Goodband (2012), *S^{10} Unified Field Theory*, url: www.mjgoodband.co.uk/articles/STUFT.pdf